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A simple procedure for calculating the mean and maximum bed stress under wave and current conditions for rough turbulent flow based on Soulsby and Clarke's (2005) method

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A B S T R A C T

In the marine environment, particularly on continental shelves, the processes of wave dissipation and sediment transport depend crucially on the bed shear stress. When waves and currents are superimposed the bed shear stress exhibits non-linear behaviour such that it is insufficient to simply sum vectorially the wave-alone and current-alone stress components. Soulsby and Clarke (2005) [Bed shear-stresses under combined waves and currents on smooth and rough beds. Report TR 137. HR Wallingford, Wallingford, UK, 22 pp.] developed a simple analytical, non-iterative method for calculating the mean and maximum bed shear stress in combined wave and current flows. For rough turbulent flow their method produces non-linearities in the mean and maximum bed stress that are consistent with those measured, while it under-predicts the non-linearities in the bed stresses in comparison with model predictions. Here their method is generalised such that it may be used to predict non-linearities in the bed stresses that are consistent with either the measurements or the models, for the case of rough turbulent flow. Also the Soulsby and Clarke method relies on the fact that the depth-averaged current is already known. However, in the field it is often the case that the current at a particular height above the bed is measured, so here their method is reposed in terms of a current measured at a particular height. A MATLAB script is provided for this modified Soulsby and Clarke method, such that either strong or weak non-linearity can be included in the wave–current interaction and the current can be input at a particular height or as a depth average.

1. Introduction

On the marine continental shelf, an understanding of the mean and maximum bed shear stresses that occur under waves and currents is important in the processes of wave propagation and sediment transport. As waves propagate over the continental shelf the bottom friction that they 'feel' can be a significant energy 'sink'. This, in turn, can influence mean current strength, particularly in shallower areas of the continental shelf seas. In sediment transport studies, the maximum bed shear stress controls the grain size that can get into suspension, while the maximum and the wave-enhanced, mean bed shear stress control the shape of the concentration profile.

Waves and currents interact non-linearly in the wave bottom boundary layer (WBL) such that the mean, and probably also the maximum, bottom stresses are not the same as given by a linear, vector sum of the bed stresses for an equivalent wave and depth-averaged current in isolation. There has been extensive literature on this subject based on experimental (Kemp and Simons, 1983; Arnskov et al., 1993; Simons et al., 2000; Musumeci et al., 2006) and field (Cacchione and Drake, 1982; Huntley and Hazen, 1988; Trowbridge and Agrawal, 1995; Styles, 2006) measurements as well as analytical (Grant and Madsen, 1979; Christoffersen and Jonsson, 1985; Myrhaug and Slaattelid, 1989; Madsen and Wikramanayake, 1991; Malarkey and Davies, 1998) and numerical (Fredsøe, 1984; Davies, 1990; Antunes do Carmo et al., 2003) models. Soulsby et al. (1993) compared non-linearity as predicted by these detailed process models with available data. This showed that the models tended to produce more non-linearity than observed, particularly in the maximum bed shear stress.

The Soulsby and Clarke (2005) method, hereafter referred to as SC05, is a practical formulation that relies on standard input quantities such as the depth-averaged current, wave velocity amplitude, water depth and bed roughness in order to calculate the mean and maximum bed shear stress for both rough and smooth turbulent flow associated with skin friction. It distinguishes itself by representing the non-linearity in the simplest way and also by reflecting the non-linearity seen in the data that

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has thus far been collected. It is based on a physical argument such that, like other analytical models, it can be used to produce the vertical profile of the current. Unlike other analytical models, the method does not require any iteration.

The primary aim of this technical note is to generalise the SC05 method for rough turbulent flows by including the possibility of greater non-linearity consistent with detailed process models. This is done without attempting to assess the validity of the various models, but rather to allow the user to investigate the potential effect of the strong non-linearity that is posed by most such models. The secondary aim of this note is to repose the method in terms of a current at a particular height rather than the depth-averaged current, since the latter is not always known (e.g. Bolaños et al., 2012).

The note begins with an outline of the SC05 method which allows the calculation of the mean and maximum bed stress from standard inputs including the depth-averaged current. This includes the new generalisation of the method to allow more non-linearity to be included. The degree of non-linearity in this modified SC05 method is compared with other models of the wave–current interaction. The extra procedure to calculate the depth-averaged current is then explained together with an example case to demonstrate it. Finally the discussions and outcomes of the study are presented. The note also provides a MATLAB script and documentation of the example case discussed later.

2. The SC05 method

The SC05 method seeks to find the magnitudes of the mean and maximum (kinematic) bed shear stresses, τ_m and τ_{max}, that result when waves and currents are superimposed at some angle, φ, to one another, as shown schematically in Fig. 1(a), where τ_p is the maximum of the periodic part of the stress associated with the wave. The method requires the following input quantities:

$$h, \quad z_0, \quad u_w, \quad T, \quad \langle u \rangle, \quad \varphi, \tag{1}$$

where h is the water depth, z_0 is the seabed roughness, u_w is the near-bed wave velocity amplitude, T is the wave period and $\langle u \rangle$ is the depth-averaged current.

In the absence of advective terms and in the steady state the cycle mean (kinematic) shear stress profile τ_{mz}, for waves and currents superimposed at any angle to one another, is given by $\tau_{mz}=\tau_m(1-z/h)$, where z is the vertical coordinate, $\tau_m = {u_{*m}}^2$, u_{*m} is the mean shear velocity. Like the earlier approach of Christoffersen and Jonsson (1985), hereafter CJ85, SC05 separated the wave and current parts of the motion by assuming a time-invariant, turbulent eddy viscosity, ν_t, of the form

$$\nu_t = \begin{cases} \kappa u_{*e} z, & z_0 < z \le \delta, \\ \kappa u_{*m} z(1-z/h), & \delta < z \le h, \end{cases} \tag{2}$$

See Fig. 1(b), where κ is the von Kármán constant, $\kappa=0.4$, u_{*e} is the effective friction velocity, δ is the WBL thickness, $\delta=0.24u_{*w}/\omega$ and $\omega=2\pi/T$. Here δ is defined in terms of the wave-alone friction velocity, u_{*w}, rather than the maximum combined friction velocity as in other time-invariant eddy viscosity models, and the coefficient (0.24) was chosen by SC05 on the basis of laboratory data rather than 0.367κ (CJ85) or 2κ (Grant and Madsen, 1979; hereafter GM79). The effective friction velocity in Eq. (2) is given by $u_{*e}=\tau_e^{1/2}$, where τ_e is the magnitude of the combined effective bottom shear stress

$$\tau_e = \sqrt{\tau_c^2 + \tau_w^2}. \tag{3}$$

Here τ_c is the magnitude of the current-alone kinematic bed shear stress, $\tau_c = {u_{*c}}^2$, u_{*c} is the current-alone friction velocity, ${u_{*c}}^2 = C_D\langle u \rangle^2$, τ_w is the magnitude of the wave-alone maximum kinematic bed shear stress, $\tau_w = {u_{*w}}^2 = f_w u_w^2/2$, $C_D=\kappa^2/\log^2(h/z_0 e)$ is the drag coefficient for the current alone (log is to the base e), $f_w=1.39(a_w/z_0)^{-0.52}$ is the friction factor for waves alone (Soulsby, 1997) and a_w is the near-bed wave orbital excursion, $a_w=u_w/\omega$. In relation to Fig. 1, τ_c is analogous to τ_m and τ_w is analogous to τ_p but in general because of non-linearity $\tau_c \ne \tau_m$ and $\tau_w \ne \tau_p$. SC05 used $\tau_{mz}=\nu_t \mathrm{d}u/\mathrm{d}z$ together with Eq. (2) to show that, like the vertical velocity profile of GM79, under wave–current conditions the current, $u=u(z)$, is described by a two-stage, logarithmic profile

$$u = \begin{cases} \dfrac{{u_{*m}}^2}{\kappa u_{*e}} \log(z/z_0), & z_0 < z \le \delta, \\ \dfrac{{u_{*m}}^2}{\kappa u_{*e}} \log(\delta/z_0) + \dfrac{u_{*m}}{\kappa}\log(z/\delta), & \delta < z \le h, \end{cases} \tag{4}$$

See Fig. 1(c). In Eq. (4), it has been assumed that $\tau_{mz}=\tau_m$ for $z \le \delta$. When $z > \delta$, the velocity can alternatively be expressed as $u=\kappa^{-1}u_{*m}\log(z/z_A)$ where z_A is the apparent roughness that the outer flow 'feels' as a result of the added wave mixing and is given by $z_A = z_0^\gamma/\delta^{\gamma-1}$, where $\gamma=u_{*m}/u_{*e}$. In the wave-alone case Eq. (4) reduces to $u=0$ since $u_{*m}=0$ and in the current-alone case $u_{*m}=u_{*c}$ and $\delta=z_0$ so that Eq. (4) reduces to $u=\kappa^{-1}u_{*c}\log(z/z_0)$. However unlike the GM79 model, and other time-invariant eddy viscosity models, u_{*e} and δ are dependent only on current-alone and wave-alone quantities (u_{*c}, u_{*w}) rather than combined quantities (u_{*m}, u_{*_p}). This means that while other time-invariant eddy viscosity models require an iteration to find a solution, the SC05 method does not. The only unknown in Eq. (4) for the SC05 method is u_{*m}. Thus SC05 showed that when $z_0 \ll \delta \ll h$, $\langle u \rangle$

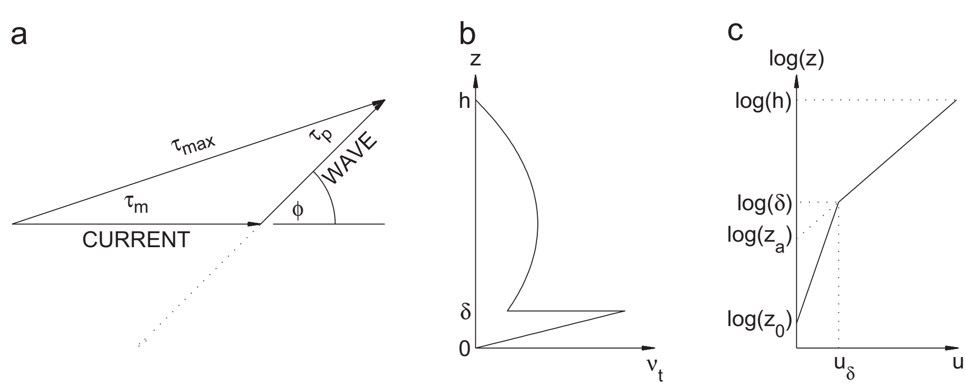

Fig. 1. Definition sketches of (a) the bed stresses for waves and currents at an angle to one another; (b) the turbulent eddy viscosity profile given by Eq. (2); and (c) two-stage logarithmic profile given by Eq. (4).