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Optimal dividends in the dual model

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Abstract

The optimal dividend problem proposed by de Finetti [de Finetti, B., 1957. Su un'impostazione alternativa della teoria collettiva del rischio. In: Transactions of the XVth International Congress of Actuaries, vol. 2. pp. 433–443] is to find the dividend-payment strategy that maximizes the expected discounted value of dividends which are paid to the shareholders until the company is ruined or bankrupt. In this paper, it is assumed that the surplus or shareholders' equity is a Lévy process which is skip-free downwards; such a model might be appropriate for a company that specializes in inventions and discoveries. In this model, the optimal strategy is a barrier strategy. Hence the problem is to determine b^*, the optimal level of the dividend barrier. A key tool is the method of Laplace transforms. A variety of numerical examples are provided. It is also shown that if the initial surplus is b^*, the expectation of the discounted dividends until ruin is the present value of a perpetuity with the payment rate being the drift of the surplus process.

Keywords: Barrier strategies; Optimal dividends; Dual model; Compound Poisson process; Gamma process; Subordinator; Lévy process; Smooth pasting

1. Introduction

The optimal dividends problem goes back to de Finetti (1957). To make the problem tractable, he assumed that the annual gains of a stock company are independent and identically distributed random variables that take on only the values -1 and $+1$. How should dividends be paid to the shareholders, if the goal is to maximize the expectation of the discounted dividends before possible ruin of the company?

In Bühlmann (1970), the problem is analyzed in the continuous time model of collective risk theory. In the absence of dividends, the surplus of a company at time t is

$$U(t) = u + ct - S(t), \quad t \geq 0. \tag{1.1}$$

Here, $u \geq 0$ is the initial surplus, and c is the constant rate at which the premiums are received. The aggregate claims process $\{S(t)\}$ is assumed to be a compound Poisson process.

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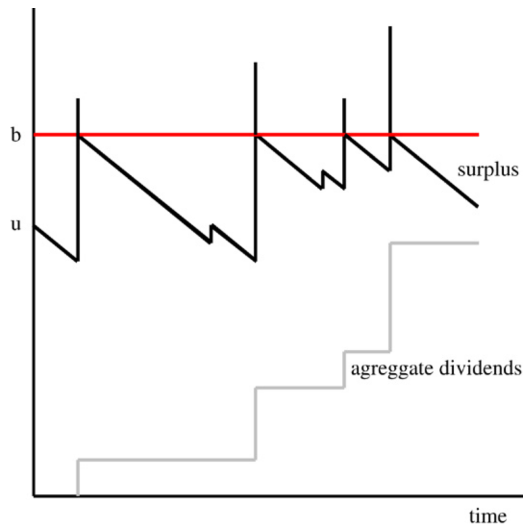

Fig. 1. Dividends and surplus under a barrier strategy.

The purpose of this paper is to examine the *dual problem*, where the surplus or equity of the company (in the absence of dividends) is of the form

$$U(t) = u - ct + S(t), \quad t \geq 0. \tag{1.2}$$

Here u is again the initial surplus, but the constant c is now the rate of expenses, assumed to be deterministic and fixed. The process $\{S(t)\}$ is a compound Poisson process, given by the Poisson parameter λ and the probability density function $p(y)$, $y > 0$, of the positive gains. In this model, the expected increase of the surplus per unit time is

$$\mu = \mathrm{E}[S(1)] - c = \lambda \int_0^\infty y p(y) \mathrm{d}y - c. \tag{1.3}$$

It is assumed to be positive.

Whereas a model of the form (1.1) is appropriate for an insurance company, a model of the form (1.2) seems to be natural for companies that have occasional gains whose amount and frequency can be modelled by the process $\{S(t)\}$. For companies such as pharmaceutical or petroleum companies, the jump should be interpreted as the net present value of future income from an invention or discovery. Other examples are commission-based businesses, such as real estate agent offices or brokerage firms that sell mutual funds or insurance products with a front-end load. Postulating that the model might be appropriate for an annuity or pension fund, some authors have derived ruin probability results; see Cramér (1955, Section 5.13), Seal (1969, pp. 116–119), Tákacs (1967, pp. 152–154), and the references cited therein.

Assuming a barrier strategy, we begin by defining a function for the expected value of discounted dividends until ruin. Before displaying general results on the optimal dividend strategy in Section 5, two specific examples of this function are given in Sections 3 and 4. In Section 6, an alternative approach is introduced and developed for the case where the jump amounts follow a mixture of exponential distributions. With the help of Laplace transforms, Section 7 expands this method for any type of jump distribution. Numerical illustrations are displayed in Section 8. Finally, the method is generalized to every process $\{S(t)\}$ with independent, stationary, and nonnegative increments.

2. Barrier strategies

It is assumed that dividends are paid according to a barrier strategy. Such a strategy has a parameter $b > 0$, the level of the barrier. Whenever the surplus exceeds the barrier, the excess is paid out immediately as a dividend. This is illustrated in Fig. 1. Note that dividend amounts are discrete.

Remark 2.1. In the case of commission-based gains, this dividend policy is straightforward. When the jump amounts are to be interpreted as the net present value of future income, the excess over the barrier may not be distributed as