Available online at www.sciencedirect.com
ScienceDirect
Insurance: Mathematics and Economics 40 (2007) 95–103
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/ime

Joint distributions of some actuarial random vectors in the compound binomial model[☆]

Guoxin Liu[a,*], Jinyan Zhao[a,b]

[a] *School of Sciences, Hebei University of Technology, Tianjin 300130, China*
[b] *School of Sciences, Shenyang University of Sciences and Technology, Shenyang 110168, China*

Received March 2005; received in revised form March 2006; accepted 3 March 2006

Abstract

The compound binomial model is first proposed by Gerber [Gerber, H.U., 1988. Mathematical fun with compound binomial process. Astin Bull. 18, 161–168]. In this paper, we introduce a renewal mass function of a defective renewal sequence constituted by the up-crossing zero points of the model and get its explicit expression. By the mass function together with the strong Markov property of the surplus process $\{X(n)\}$, we obtain the explicit expressions of the ruin probability, the joint distribution of T, $X(T-1)$ and $|X(T)|$ (i.e., the time of ruin, the surplus immediately before ruin and the deficit at ruin) and distributions of some actuarial random vectors containing more than three variables.

Classification code: IM11; IM13

Keywords: Compound binomial model; Joint distribution; Renewal mass function; Ruin probability; Sequence of up-crossing zero points; Ultimately leaving deficit time

1. Introduction

Let us first introduce the compound binomial model. The surplus process is

$$X(n) = u + n - \sum_{k=1}^{N(n)} U_k,$$

for $n = 0, 1, 2, \ldots$. Here $X(0) = u$ is the initial capital, the periodic premium is one and the claim sequence $\{U_k\}$ is assumed to be independent and identically distributed random variables with common purely discrete distribution

$$P(U = k) = p_k, \quad k = 1, 2, \ldots, \tag{1}$$

☆ This work has been supported by The Hebei Natural Science Foundation (Grant No.103001).

* Corresponding author. Tel.: +86 22 2656 4469; fax: +86 22 2656 4469.
E-mail address: gxliu@hebut.edu.cn (G. Liu).

0167-6687/$ - see front matter
doi:10.1016/j.insmatheco.2006.03.001

with $\sum_{k=1}^{\infty} p_k = 1$ and a finite expectation μ_U. $N(n)$, the number of claims arrived up to time n is assumed to be binomially distributed,

$$P(N(n) = k) = C_n^k p^k (1-p)^{n-k}, \quad k = 0, 1, 2, \ldots. \tag{2}$$

Here, in the remaining part of this paper, $C_n^k := \frac{n!}{(n-k)!k!}$. Denote the inter-claim times by $\{W_n\}_1^{\infty}$, then they are i.i.d. with geometric distribution

$$P(W_n = k) = p(1-p)^{k-1}, \quad n, k = 1, 2, \ldots.$$

And $\{N(n)\}$ is independent of $\{U_k\}$ in this case.

Assume that the *net profit condition* is $p\mu_U < 1$. Under the net profit condition we have

$$P\left(\lim_{n\to\infty} X(n) = \infty\right) = 1. \tag{3}$$

Let T be the time of ruin and L the time of the surplus process leaving deficit ultimately (simply, it is called the ultimately leaving deficit time), i.e.

$$T = \inf\{n \geq 0 : X(n) < 0\}, \qquad L = \sup\{n \geq 0 : X(n-1) = -1, X(n) = 0\} \tag{4}$$

($\inf \emptyset = \infty$ and $\sup \emptyset = 0$ for convention). From (3) and (4), we see that $P(L < \infty) = 1$. Denote the ruin probability and the survival probability with the initial capital u by $\psi(u) = P(\tau(u) < \infty)$ and $\phi(u) = 1 - \psi(u)$ respectively.

The compound binomial model was first proposed by Gerber (1988). It is a discrete time analogue of the compound Poisson model. As pointed out by Cheng et al. (2000), with a relatively simple method, attractive results can be derived in this model. And, in fact, the latter can be viewed as a limiting case of the former (see, for example, Liu et al. (2005)). It has been extensively studied in the literature. For example, see Shiu (1989), Willmot (1993), Dickson (1994), Cheng et al. (2000) and references therein.

Besides ruin probability, some more actuarial quantities, such as the time of ruin, the surplus immediately before ruin and the deficit at ruin, are of interests in ruin theory. See, for example, Gerber et al. (1987), Gerber and Shiu (1997), Wei and Wu (2002), Wu et al. (2003) and references therein. Among them, Gerber and Shiu (1997) studied the joint distribution of the three important actuarial quantities: the time of ruin, the surplus immediately before ruin and the deficit at ruin for the compound Poisson model. Motivated by Gerber and Shiu (1997), Wu et al. (2003) obtained the explicit expression of this joint distribution and the joint distributions of some actuarial random vectors for the same model by virtue of a renewal mass function, which can also be expressed explicitly.

The aim of this paper is to derive the corresponding joint distributions in the compound binomial model along the same lines in Wu et al. (2003). In Section 2, we begin with the construction of a renewal sequence in the compound binomial model and present the explicit expression of the renewal mass function for the renewal sequence. By the way, the explicit expression of the ruin probability is obtained in terms of the renewal mass function. Due to the special setting of the model, the sequence of zero points in Wu et al. (2003) is substituted by the sequence of up-crossing zero points defined in Section 2. In Section 3, the explicit expressions of the joint distributions of some actuarial quantities are presented by the renewal mass function, such as $(T, X(T-1), |X(T)|)$, $(T, X(T-1), |X(T)|, \inf_{0\leq n<\tau_1} X(n))$ and $(T, X(T-1), |X(T)|, \inf_{0\leq n<\tau_1} X(n), \inf_{\tau_1\leq n<L} X(n))$, where τ_1 is the first up-crossing zero point defined in the next section. Hence, $-\inf_{0\leq n<\tau_1} X(n)$ is the maximal deficit from ruin to recovery and $-\{\inf_{0\leq n<\tau_1} X(n) \wedge \inf_{\tau_1\leq n<L} X(n)\}$ is the maximal deficit as a whole.

2. Up-crossing zero points and renewal mass function

Define the sequence of up-crossing zero points $\{\tau_k\}_{k\geq 1}$ recursively by

$$T_1 = T, \qquad \tau_1 = \inf\{n > T_1 : X(n) = 0\};$$
$$T_k = \inf\{n > \tau_{k-1} : X(n) < 0\}, \qquad \tau_k = \inf\{n > T_k : X(n) = 0\}, \quad k = 2, 3, \ldots.$$

If the insurance company never stops its activities whenever it is in the red or not, T_k can be viewed as the kth ruin time and τ_k the kth recovery time. From the definition, we can see that $X(\tau_k - 1) = -1$ and $X(\tau_k) = 0$ for each k. This is the reason why we call $\{\tau_k\}_{k\geq 1}$ the sequence of up-crossing zero points. Let $\mathcal{F}_n = \sigma\{X(k), k \leq n\}$ for