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# Minimax pricing and Choquet pricing

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**Abstract**

The Choquet pricing and minimax pricing, which are nonlinear expectations, have been widely used in economics, finance and insurance as an alternative to traditional mathematical expectation. However, it is usually not easy to calculate these due to their nonlinearity. In this paper, we consider the calculation of a class of Choquet expectations and minimax expectations obtained from the pricing of a contingent claim with multiple prior probability measures. We show that both the Choquet pricing and minimax pricing of some European options are same, although this result is not in general true for non-European options.
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## 1. Introduction

Sets of probability measures arise in statistics in the areas of robustness, group decision making and insurance. One method of measuring risk is to find extreme probabilities over a set of possible or reasonable probability measures. This idea gives rise to upper and lower probability measures which are usually considered as convex (concave) capacities. Choquet expectations were introduced by Choquet (1953). Upper and lower probability measures and the associated Choquet expectations and minimax expectations are used extensively in the robustness literature; see, for example, Derobertis and Hartigan (1981), Sarin and Wakker (1998), Huber and Strassen (1973), Wakker (2001), Wasserman and Kadane (1990). Choquet expectations and minimax expectations have been introduced in finance and insurance as an alternative to traditional mathematical expectation; see, for example, De Waegenaere and Wakker (2001), Wang (2000), Wang and Dhaene (1998), Wang et al. (1997), Young (1999), De Waegenaere et al. (2003). With Choquet pricing, the pricing of an insurance or contingent claim equals the Choquet integral of the corresponding payoff with respect to a convex (concave) sub-additive measure generated by a set of probability measures. With minimax pricing, the pricing of an insurance or contingent claim equals the maximal (minimal) expectations with respect to a set of probability measures. Choquet expectation and minimax expectation have several properties that make them especially suitable for the pricing in insurance contract or financial asset. In general, it is not easy to calculate these due to their nonlinearity. In this paper, we consider the calculation of a class of Choquet expectations which arise from the pricing

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of European option with multiple prior probability measures. The set of probability measures considered is a natural family in this application, but the associated upper and lower probability measures are no longer concave (convex).

More precisely, let $\{S_t\}$ be the price of a stock evolving as a stochastic differential equation

$$\mathrm{d}S_t = \mu_t S_t \mathrm{d}t + \sigma_t S_t \mathrm{d}W_t$$

where $\{\mu_t\}$ and $\{\sigma_t\}$ are two adapted processes. The Black–Scholes theory and Girsanov's formula states that there exists a unique risk neutral martingale measure $Q$ defined by

$$\frac{\mathrm{d}Q}{\mathrm{d}P} = e^{-\frac{1}{2}\int_0^T \left(\frac{\mu_s - r}{\sigma_s}\right)^2 \mathrm{d}s + \int_0^T \left(\frac{\mu_s - r}{\sigma_s}\right) \mathrm{d}W_s}$$

where $Q$ depends on the parameter $\mu$ such that for any contingent claim $\xi$ at time $T$, the pricing of $\xi$ is given by $E_Q[\xi e^{-rT}]$, where $r$ is the interest rate of a bond. However, in practice we sometimes do not know the real value of $\mu$ or $\sigma$. Suppose the value of $\mu$ is known only imprecisely, and more specifically that $\mu_t \in [r - k\sigma_t, r + k\sigma_t]$ for a given positive constant $k > 0$. Let $v_t := \frac{\mu_t - r}{\sigma_t}$. The prior risk neutral martingale measure is no longer unique, but it belongs to the set

$$\mathcal{P} = \left\{ Q^v : \frac{\mathrm{d}Q^v}{\mathrm{d}P} = e^{-\frac{1}{2}\int_0^T |v_s|^2 \mathrm{d}s + \int_0^T v_s \mathrm{d}W_s}, \sup_{t \in [0,T]} |v_t| \le k \right\}. \tag{1}$$

In this case, there are two ways to model the super-pricing and sub-pricing of a contingent claim. One is the minimax pricing method; see, for example, El Karoui et al. (1997)

$$\bar{\mathcal{E}}[\xi] = \sup_{Q \in \mathcal{P}} E_Q[\xi]; \qquad \underline{\mathcal{E}}[\xi] = \inf_{Q \in \mathcal{P}} E_Q[\xi].$$

Here, we use the notation $E_Q$ to denote expectation with respect to the probability measure $Q$. The other method is to use Choquet pricing; see, for example, Wang (2000), De Waegenaere et al. (2003). It is defined by

$$\bar{C}[\xi] = \int_{-\infty}^0 (V_1(\xi \ge t) - 1)\mathrm{d}t + \int_0^\infty V_1(\xi \ge t)\mathrm{d}t,$$

$$\underline{C}[\xi] = \int_{-\infty}^0 (V_2(\xi \ge t) - 1)\mathrm{d}t + \int_0^\infty V_2(\xi \ge t)\mathrm{d}t,$$

where $V_1$ and $V_2$ are upper and lower probabilities defined by

$$V_1(A) = \sup_{Q \in \mathcal{P}} Q(A) \quad \text{and} \quad V_2(A) = \inf_{Q \in \mathcal{P}} Q(A).$$

Obviously,

$$\underline{C}[\xi] \le \underline{\mathcal{E}}[\xi] \le \bar{\mathcal{E}}[\xi] \le \bar{C}[\xi].$$

A natural question is the following: under which conditions for European option or contingent claim $\xi$ do the minimax and Choquet methods give the same upper and lower bound prices? That is under what conditions are two pairs

$$\bar{\mathcal{E}}[\xi] = \bar{C}[\xi], \qquad \underline{\mathcal{E}}[\xi] = \underline{C}[\xi] \tag{2}$$

equal? Chen et al. (2005) have shown that (2) holds for all random variables $\xi$ with finite second moments if and only if $\mathcal{P}$ has exactly one element, that is if and only if $k = 0$. Thus if $k > 0$ the equalities (2) in general do not hold for all $\xi$ with finite second moments. However, in this paper, we shall show the above equalities (2) are true for some European options, in particular options whose payoff is a monotone function of the terminal stock price; see the main result given in Theorem 1. The proof relies on some properties of backward stochastic differential equations (BSDEs). We also give some examples to illustrate how to calculate the Choquet price by using our methods.

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