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Analysis of risk measures for reinsurance layers

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Abstract

We analyze common risk measures for reinsurance layers defined in terms of lower and upper retentions. In particular, we consider the Value-at-Risk, the variance, the coefficient of variation, the dispersion and the reduction effect. In a first part, we compute some risk measures for a general layer. In a second part, we compare several risk measures among the different layers in a reinsurance chain.

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1. Introduction

Let $\{Y_i;\ i \geq 1\}$ be a sequence of successive claim sizes consisting of independent and identically distributed random variables generated by the distribution function F_Y of a nonnegative random variable Y. Let N be a nonnegative integer-valued random variable, independent of the Y_i's, representing the number of claims occurring in some fixed time interval. We denote by $Y_1^*, Y_2^*, \ldots, Y_N^*$ the order statistics, arranged in increasing order, of the random sample $Y_1, Y_2, \ldots, Y_N$ of successive claim sizes in the time interval.

Reinsurance can be considered as one way of risk sharing. The reinsurance forms all have in common the intention to diminish an excessive number of claims and/or the impact of the large claims. Of course, reinsurance diminishes the volatility in the portfolio as the risk is shared between the first line insurance and the reinsurance. The decision to involve other partners in the risk sharing depends on many factors, some of them have only marginal relation with reinsurance.

A first line insurance will always try to safeguard its position by subscribing itself to a variety of insurance contracts with an equally varied set of (re)insurance companies. As such, the first line insurance itself becomes an insured client by paying a specific premium to a reinsurance company in exchange for a policy covering the reinsured quantity. For the first line insurance company, it obviously does not make sense to sell the entire portfolio to a reinsurance company

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because it will then lose all premium income from that portfolio. The first line company has to ponder how it wants the claims to be split between itself and the reinsurer.

It is common to distinguish between two types of reinsurance: proportional and nonproportional. Within the area of proportional reinsurance treaties, we have two traditional forms: *quota-share* reinsurance where the reinsurer accepts a proportion $a \in (0, 1]$ of the claims experience of the total portfolio and *surplus* reinsurance where the reinsured amount is also determined by the value of the insured object as long as it exceeds a retention L. Within the framework of nonproportional reinsurance treaties, we cite four forms. An *excess-of-loss* reinsurance is determined by a retention M indicating that the reinsurer covers the part of the claims that overshoots M. In a *stop-loss* reinsurance contract, the reinsured amount is the part of the insurer's total loss overshooting a retention C. Note that excess-of-loss and stop-loss reinsurance treaties are equivalent when a single risk comes into play. Furthermore, there are reinsurance forms classified as large claims reinsurance since they are defined in terms of the order statistics of the claim sizes. A first form is *largest claims* reinsurance where the reinsured amount combines the values of the r largest claim sizes in the portfolio. A second and slightly more popular form is *ECOMOR* reinsurance which is defined as an excess-of-loss treaty with the $(r + 1)$th largest claim size as random retention. We refer to Teugels (1985), Embrechts et al. (1997) and Ladoucette and Teugels (2006) for asymptotic problems pertaining to ECOMOR as well as to largest claims reinsurance. For an overview of most of the currently employed reinsurance forms with some of their properties, see Rolski et al. (1999) and Teugels (2003).

In accordance with current practice, quota-share and stop-loss treaties are somewhat popular if the number of claims is large. Also, surplus and excess-of-loss reinsurance are more traditional if the claim sizes are large. Furthermore, reinsurance based on the largest claims is almost never used. This fact is rather surprising if reinsurance is meant to protect the insurer against large claims since, in such a case, it looks almost necessary to use a reinsurance treaty that is based on these largest claims.

Combinations of different forms of treaties are easily constructed. Schmitter (1987) combines quota-share and stop-loss treaties such that the reinsured amount is:

$$R_{a,C} := \max\left(0, a\sum_{i=1}^{N} Y_i - C\right).$$

Quota-share and excess-of-loss treaties are combined in Centeno (1985). The reinsured amount then has the following expression:

$$R_{a,M} := \sum_{i=1}^{N} \max(0, aY_i - M).$$

Benktander and Ohlin (1967) combine a surplus treaty with an excess-of-loss treaty. The reinsured amount is then:

$$R_{L,M} := \sum_{i=1}^{N} \max\left(0, \left(\frac{V_i - L}{V_i}\right) Y_i \mathbb{1}_{\{V_i > L\}} - M\right)$$

where V_i is the value insured for the ith claim policy. Here and throughout the paper, $\mathbb{1}_A$ stands for the indicator function of the event A.

Such combinations are referred to as *partial reinsurance*. For example, see Steenackers and Goovaerts (1992). Even more popular is the combination of a stop-loss treaty on top of an excess-of-loss treaty. In this case, one has for the reinsured amount:

$$R_{M,C} := \max\left\{0, \sum_{i=1}^{N} \max(0, Y_i - M) - C\right\}.$$

A further generalization is called *drop down excess-of-loss* reinsurance. In this type of setting, the claim size is curtailed at both ends, both of them depending on the order of the claim. The reinsured amount has a form of the kind:

$$R_{M_i,L_i} := \sum_{i=1}^{N} \min\{L_i, \max(0, Y^*_{N-i+1} - M_i)\}$$