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Distribution-free option pricing

Ann De Schepper*, Bart Heijnen

University of Antwerp, Faculty of Applied Economics, Prinsstraat 13, B-2000 Antwerp, Belgium

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Abstract

Nobody doubts the power of the Black and Scholes option pricing method, yet there are situations in which the hypothesis of a lognormal model is too restrictive. A natural way to deal with this problem consists of weakening the hypothesis, by fixing only successive moments and possibly the mode of the price process of a risky asset, and not the complete distribution. As a consequence of this generalization, the option price is no longer a unique value, but rather a range of possible values. In the present paper, we show how to find upper and lower bounds for this range, a range which turns out to be quite narrow in a lot of cases.

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1. Introduction

Since the famous paper of Black and Scholes (1973), the problem of how to determine prices for basic options has been solved at least to a certain extent. Many publications since then still deal with the same problem, mainly in two directions: firstly, the extension of their model to more complex classes of options, and secondly, alternatives or sophistications of their formula taking into account perceived deviations from the lognormal process, such as skewness, excess kurtosis, jumps and extreme events.

In the present paper, we want to make a contribution to this last category, by showing how to price options without imposing a complete model on the underlying price process – as is the case for the pricing formula of Black and Scholes. The main reason for our approach is the observation of several authors in the past, that – although the lognormal model, which makes up the foundation of the formula of Black and Scholes, is usually a rather good model for describing real price processes – there are some shortcomings of the model that can become important, and that can cause (more or less seriously) biased option prices. Without claiming any exhaustivity, we can cite e.g. interesting contributions of Teichmoeller (1971), Hull and White (1988), Becker (1991), Bakshi et al. (1997), Corrado and Su (1998), Gerber and Landry (1998), Sarwar and Krehbiel (2000), Backus et al. (2002), Kou (2002) and Gençay and Salih (2003).

* Corresponding author. Tel.: +32 3 220 40 77; fax: +32 3 220 48 17.
E-mail address: ann.deschepper@ua.ac.be (A. De Schepper).

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The method we present in this paper only utilizes successive moments of the underlying price process under a risk-neutral probability measure, and not the distribution itself. Thus, the hypothesis used to reach an option price is much weaker than is the case for the Black and Scholes formula. As a consequence, one unique value for the option price is not possible, since the knowledge of successive moments only gives limited information about the real process, such that expected pay-offs can only be calculated approximately. However, we will show that it is possible to construct close (and in some cases very close) absolute upper and lower bounds for the options prices, which can also be combined to yield one approximate price.

The paper is organized as follows. We start in Section 2 with a description of the problem and of the methodology. Afterwards, in Section 3, we present the results for bounds on European option prices when limited information about the price process is available. Proofs of these results are provided in the appendix. Section 4 is meant to illustrate our results numerically and graphically. Afterwards in Section 5, we show how the results can be extended to arithmetic Asian options. Section 6 concludes.

2. Description of the problem

2.1. The pricing of European call options

Consider a European call option on a risky asset with current price S, that matures at time T having exercise price K. In an arbitrage-free setting, the price of this option can be determined as

$$B_{EC}(T, K, S) = \mathrm{e}^{-rT}\ \mathbb{E}^{Q}\left[(S_T - K)_+\right], \tag{1}$$

where r denotes the risk-free interest rate, and where the stochastic process $\{S_t, t \geq 0\}$, starting at $S_0 = S$, describes the price process of the underlying risky asset. We assume that Q is the unique equivalent probability measure, such that the discounted price process is a martingale, or $\mathbb{E}^{Q}\left[\mathrm{e}^{-rt} S_t\right] = S$.

Following Merton (1973), we know that the option price must satisfy

$$\max(0, S - \mathrm{e}^{-rT} K) \leq B_{EC}(T, K, S) \leq S, \tag{2}$$

whatever the real price process of the underlying asset.

An exact option price can only be computed, if the distribution of the price process $\{S_t, t \geq 0\}$ is known for sure — which in reality is not the case. The most commonly used assumption is the Black and Scholes setting, where the price process is assumed to follow a geometric Brownian motion. This means that

$$S_t = S \cdot \mathrm{e}^{(r - \frac{1}{2}\sigma^2)t + \sigma W_t}, \tag{3}$$

where $\{W_t, t \geq 0\}$ is a standard Brownian motion. Thus, under the measure Q, the variables S_t/S are lognormally distributed with mean $(r - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$. Under these assumptions, the price of a European call can be found according to the well-known Black and Scholes formula (see Black and Scholes (1973)), or

$$B_{EC}^{(B\&S)}(T, K, S) = S \cdot \Phi(d_1) - K\mathrm{e}^{-rT} \cdot \Phi(d_2), \tag{4}$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sqrt{\sigma^2 T}}\left(\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right) T\right) \\ d_2 &= \frac{1}{\sqrt{\sigma^2 T}}\left(\ln(S/K) + \left(r - \frac{1}{2}\sigma^2\right) T\right), \end{aligned} \tag{5}$$

where $\Phi : \mathbb{R} \to [0, 1]$ denotes the cumulative distribution function of the standard normal distribution.

As mentioned in the introduction, the Black and Scholes pricing formula shows some imperfections in that the whole distribution of the price process is fixed by means of a lognormal model. Although this model performs well in a lot of cases (it is still the most widely used approach for the valuation of options), wrong prices can arise due to the strong assumptions, e.g. as the tails of the lognormal distribution are not as fat as in reality. In fact this problem is an aspect of the more general issue of model risk, a phenomenon which in the financial literature is often a subject of