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Hedging guarantees in variable annuities under both equity and interest rate risks

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Abstract

Effective hedging strategies for variable annuities are crucial for insurance companies in preventing potentially large losses. We consider discrete hedging of options embedded in guarantees with ratchet features, under both equity (including jump) risk and interest rate risk. Since discrete hedging and the underlying model considered lead to an incomplete market, we compute hedging strategies using local risk minimization. Our results suggest that risk minimization hedging, under a joint model for the underlying and interest rate, leads to effective risk reduction. Moreover, hedging with standard options is superior to hedging with the underlying when both equity and interest rate risks are appropriately modeled.

JEL classification: G22; G10; G11; IE43; IM22; IM20

Keywords: Variable annuity; Lookback option; Equity risk; Interest rate risk; Risk minimization

1. Introduction

Annuities are contracts designed to provide payments to the holder at specified intervals, usually after retirement. Traditionally, insurance companies offered fixed annuities which guarantee a stream of fixed payments over the life of the contract. This type of annuities was attractive to the policy holders in the context of high interest rates and high cost of investment in the equity market. However, bullish markets and low interest rate environments motivate the investors to look for higher returns than those provided by the conventional annuities. Variable annuities, whose future benefits are based on the performance of a portfolio of securities including equities, have proved to be very attractive for investors, since they not only provide participation in the stock market, but they also have some protection against the downside movements in the market. Variable annuities in the U.S. are similar to the unit-linked annuities in the U.K. and the segregated funds in Canada.

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Variable annuities are appealing to investors because they are tax-deferred and they offer different types of benefits, such as the guaranteed minimum death benefits (GMDB). Until the beginning of the 1990s, the death benefits were just simple principal guarantees (original investment) or rising floor guarantees (original investment accrued at a minimally guaranteed interest rate, possibly capped at a predetermined level). In the circumstances of the bullish market of the 1990s, insurance companies have started to offer GMDB with more attractive features, such as the ratchet, which guarantees a death benefit based upon the highest anniversary account value. The anniversary dates at which the guarantee is reset are typically annual.

The simultaneous occurrence of death and market downturn seemed unlikely during the strong bullish market of 1990s, however, in the following market crash, insurance companies realized that they may face extremely large losses. Devising good risk management strategies has become of crucial importance. The traditional actuarial methods adopt a passive strategy of holding a sufficient reserve in risk-free instruments in order to meet the liabilities with high probability. Recent research applies methods from finance for computing the fair price of a guaranteed minimum death benefit in a variable annuity and meeting the contract liabilities. The typical risk management strategies in this case consist of holding positions in stocks and bonds and dynamically rebalance these positions in order to cover the guarantees. The financial engineering approach is based on the fact that the guaranteed minimum death benefit can be viewed as a put option with a stochastic maturity date. This put option has a strike equal to the initial investment for a GMDB with principal guarantee, or a strike increasing at the minimum guaranteed rate in the case of a rising floor feature. For a GMDB with ratchet features, the corresponding option is a lookback put for which the strike price is equal to a running maximum of the account value.

Brennan and Schwartz (1976), Boyle and Schwartz (1977), Aase and Persson (1992), Persson (1993), and Bacinello and Ortu (1993a) use option theory to price and hedge the embedded options in variable annuities. With the main assumption that the market is complete under both financial and mortality risk, the option price is equal to the expected value of the payoff with respect to a risk-neutral probability measure. Moreover, the option can be exactly replicated using delta hedging. The number of shares of the underlying held in a delta hedging strategy is given by the sensitivity (delta) of the option value to the underlying.

Typically, if the number of policyholders is large enough, it can be assumed that the market is complete under mortality risk. By the Law of Large Numbers, the total liability in this case will be close to its expected value. An insurance company can diversify away its mortality risk by selling enough policies. In this context, the embedded put options can be assumed to have a deterministic maturity. Moller (1998, 2001a,b) investigates pricing and hedging of insurance contracts under mortality risk.

Assuming market completeness under financial risk is, however, more problematic. One issue is that the benefits are sensitive to the tail distributions of the underlying accounts. While empirical market data shows that the distributions of equity returns exhibit fat tails, this behavior cannot be explained by the simple Black–Scholes model for equity prices. Unfortunately, as soon as one allows for stochastic volatility, or if a jump component is added to the model, the market becomes incomplete. Moreover, liquidity constraints and the impossibility of hedging continuously in time, coupled with the need to rebalance as little as possible due to the impact of transaction costs, also lead to an incomplete market. Another problem with modeling the life insurance contracts is that, because of the long maturities of these contracts, stochastic interest rates may be more appropriate than a constant rate.

The main emphasis of the literature has been on pricing the options embedded in the life insurance contracts; however, hedging is also very important for risk management purposes. In this paper we investigate the computation and effectiveness of hedging strategies under both equity and interest rate risks. We assume that the market is complete under mortality risk, but the financial market is incomplete, due to a suitable equity model for fat tails or to discrete hedging. We have analyzed the modeling of implied volatility risk in Coleman et al. (2004).

We remark that Bacinello and Ortu (1993b, 1994), Nielsen and Sandmann (1995), Miltersen and Persson (1999), and Bacinello and Persson (2002) also investigate stochastic interest rates; however, these authors focus on pricing and they assume a complete financial market which leads to the existence of a unique equivalent martingale measure for the equity price. In