Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Insurance Mathematics and Economics 39 (2006) 1–18
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/ime

The compound binomial model with randomized decisions on paying dividends[☆]

Jiyang Tan*, Xiangqun Yang

Department of Mathematics, Hunan Normal University, Changsha 410081, China

Received September 2005; received in revised form January 2006; accepted 16 January 2006

Abstract

Consider a discrete time risk process based on the compound binomial model. The insurer pays a dividend of 1 with a probability q_0 when the surplus is greater than or equal to a non-negative integer x. We will derive recursion formulas and an asymptotic estimate for the ruin probability, the probability function of the surplus prior to the ruin time, and the severity of ruin, etc.

MSC: 60K30; 90A46

Keywords: Compound binomial process; Ruin probability; Dividend; Recursion formulae; Discrete renewal equation; Asymptotic estimate

1. Introduction

In the compound binomial model, the number of insurance claims is governed by a binomial process $N(t)$, $t = 0, 1, 2, \ldots$. In any time period, the probability of a claim is $p, 0 < p < 1$, and the probability of no claim is $q = 1 - p$. We denote by $\xi_t = 1$ the event where *a claim occurs in the time period* $(t-1, t]$ and denote by $\xi_t = 0$ the event where *no claim occurs in the time period* $(t-1, t]$. Then $N(t) = \sum_{k=1}^{t} \xi_k$ for $t \geq 1$ and $N(0) = 0$. The occurrences of claims in different time periods are independent events. The claim amounts $X_1, X_2, X_3, \ldots$ are mutually independent, identically distributed, positive and integer-valued random variables; they are independent of the binomial process $\{N(t)\}$. Let the initial surplus be u, which is a non-negative integer. Traditionally, assume that the premium received in each time period is one. For $t = 1, 2, \ldots$, the surplus at time t is

$$U(t) = u + t - [X_1\xi_1 + X_2\xi_2 + \cdots + X_t\xi_t], \tag{1.1}$$

and $U(0) = u$. The compound binomial model has been studied by Gerber (1988); Shiu (1989); Willmot (1993); Dickson (1994); DeVylder (1996) (Chapter 10), DeVylder and Marceau (1996) (Section 2), and Cheng et al. (2000); Cheng and Zhu (2001).

☆ Supported by the National Natural Sciences Foundation of China (grant no. 10571051), and by the Specialized Research Fund for the Doctoral Program of Higher Education of China (20040542006).

* Corresponding author.
E-mail address: tanjiyang15@163.com (J. Tan).

0167-6687/$ - see front matter
doi:10.1016/j.insmatheco.2006.01.001

In this paper, we consider the compound binomial model modified by the inclusion of dividends. We assume that the insurer will pay a dividend of 1 with a probability q_0, $0 \le q_0 < 1$ in each time period if the surplus is greater than or equal to a non-negative integer x at the beginning of the period. See Gerber et al. (1997) (Chapter 9) for a similar model.

In Section 3 we will obtain the recursion formulas for the (expected discounted) *penalty function* associated with the time of ruin when the discount factor $v = 1$. The recursion formulas can also be seen as renewal equations. In Section 4 we will derive an asymptotic estimate for the penalty function. This will in Section 5 lead to asymptotic estimates for the ruin probability, the severity of ruin, the probability function of the surplus prior to the ruin time, and some other probability functions.

2. The model and preliminaries

In order to attract investment, an insurer usually devises some products with dividends as the following model.

We consider a discrete time risk process based on the compound binomial model (1.1). The surplus process is given by

$$U(t) = u + t - Z_t - S_t, \tag{2.1}$$

for $t = 0, 1, 2, \ldots$. In (2.1), the initial surplus u is a non-negative integer; S_t is defined as

$$S_t = \sum_{k=1}^{t} X_k \xi_k \tag{2.2}$$

for $t \ge 1$ and $S_0 = 0$; Z_t is defined as

$$Z_t = \sum_{k=1}^{t} \eta_k I(U(k-1) \ge x) \tag{2.3}$$

for $t \ge 1$ and $Z_0 = 0$, where x is a fixed non-negative integer, $I(A)$ is the indicator function of a set A, and η_k $(k \ge 1)$ is a series of randomized decision functions that are mutually independent, identically distributed and independent of $\{S_t\}$. In detail, we denote by $\eta_k = 1$ the event where *a dividend of 1 is paid at the time* k and denote by $\eta_k = 0$ the event where *no dividend is paid at the time* k. Assume

$$\Pr(\eta_k = 1) = q_0; \qquad \Pr(\eta_k = 0) = p_0, \tag{2.4}$$

where $0 \le q_0 < 1$ and $q_0 + p_0 = 1$. For $t \ge 1$, it follows from above that

$$U(t) = U(t-1) + 1 - \eta_t I(U(t-1) \ge x) - X_t \xi_t. \tag{2.5}$$

It is reasonable that the randomized decision functions η_k $(k \ge 1)$ are brought in to decide the periods with a dividend. This can be interpreted as follows: the return on the investment by an insurer in each time period can be regarded as being stochastic, and if the return on the investment in the *present* period is greater than a given level, then the insurer will pay a benefit of 1 to the insured. But, in model (2.1), if the surplus is smaller than x at the beginning of the *present* period, then a decision for paying a benefit of 1 will be cancelled. Certainly, the randomized decisions can also be related to some other things that occur with probability q_0 and do not occur with probability p_0 in each time period, for example natural disasters.

The model (2.1) is a sort of generalization of the classic risk model and is exactly the classic risk model if $q_0 = 0$. Hence our results in this paper include the corresponding results of the classic risk model.

The model with dividends in this paper is different to that of Gerber et al. (1997). In Gerber et al. (1997), a continuous time risk model (the compound Poisson model) was considered. The insurer continuously pays dividends when the surplus is no smaller than a given dividend bound, but the decision on paying a dividend is not stochastic as soon as the surplus is above the dividend bound. However, the discrete time model with dividends in this paper can be seen as analogous to the continuous time model in Gerber et al. (1997) in a way.

Put $X = X_1$ and let

$$p(k) = \Pr(X = k), \quad k = 1, 2, 3, \ldots \tag{2.6}$$