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Bed evolution numerical model for rapidly varying flow in natural streams

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ABSTRACT

A numerical model suitable for the reproduction of bed evolution in natural alluvial rivers and in channels of complex geometry is presented. It is based on a conservative formulation of one-dimensional shallow water equations, which includes an original treatment of the momentum equation source term. The proposed model has already been cross referenced with several test cases and experimental data found in the literature for fixed beds. In this study, however, the focus is on mobile bottom beds. The selected test cases are representative of some of the characteristic bed configurations that often occur in natural streams, and, therefore, are suitable for verifying the versatility of the model and its potential usefulness in the treatment of alluvial rivers. The MacCormack explicit finite difference scheme has been adopted for the numerical implementation. The liquid and solid phases are then solved by means of a semi-coupled procedure. A variety of bed evolution mechanisms and different water regime conditions are investigated in order to verify the model response in the cases of erosion, deposition and bed morphology evolution. The results show a good correspondence with the experimental data. Furthermore, the proposed model shows a notable numerical stability even when applied to test cases that are particularly difficult under the numerical computational profile, where the relative model of the standard formulation of the conservative balance equations shows evident numerical instability.

1. Introduction

Numerical modelling of free surface flows is one of the central topics in the field of hydraulic research and river engineering. The mathematical and numerical representation of hydraulic phenomena is a vast, multi-disciplinary and much discussed subject.

For fixed bed hydraulics, once the governing equations are known, important improvements and new ideas can be brought to the mathematical form of these equations, to the employed numerical schemes or to the way in which the system geometry is represented.

On the other hand, many aspects of mobile bed phenomena require further in-depth studies. There are many issues which should be taken into account. Some of the most important are the correct evaluation of the interactions between the liquid and solid phases, quantifying the solid discharge and estimating the system geometry evolution.

The setting up of every mobile bed numerical model needs the right combination of all these elements. Many authors have contributed to a deeper understanding of this process. Lyn (1987) demonstrated the strong coupling between liquid and solid phases in the case of rapidly varying boundary conditions and in the presence of

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critical flow conditions, providing valuable information to assist in choosing between coupled and uncoupled models. In the development of their model, Macchione and Falcone (2000) extended the coupling features for boundary conditions by writing the compatibility equations for the complete system. Simpson and Castelltort (2006) developed a coupled two-dimensional (2D) model applicable both to short time scale events and to long time scale phenomena in which the classical quasi-steady approximation can be invoked. Bhallamudi and Chaudhry (1991) developed a coupled mobile bed model suitable for rectangular channels. Many transport formulae are available for the estimation of sediment discharge. Each formula is derived for a particular sediment kind or flow regime, covering a large field of possibilities including sediment mixtures (Toro-Escobar et al., 1996), bed load and suspended transport (Ackers and White, 1973; Rijn, 1984a, b), steep channels (Smart, 1984) and alluvial rivers (Rijn, 1984c).

Aspects affecting morphological evolution have been investigated, such as the development of transversal cross-sections due to transport phenomena (Parker, 1978a, b) and the longitudinal evolution of the bottom profile (Bhallamudi and Chaudhry, 1991). Other aspects have been examined, such as bank stability analysis (Darby and Thorne, 1996a, b; Osman and Thorne, 1988a, b), transport transversal components associated with gravitational or diffusive forces (Ikeda et al., 1988), and tangential forces distribution along the wetted perimeter (Kovacs and Parker, 1994; Parker, 1978a, b).

A comprehensive study of all these aspects finally leads to the formulation of mathematical models which satisfactorily reproduce the transversal evolution of alluvial channels (Lamberti and Schippa, 1996; Schippa, 1993).

Considering the classical form of shallow water equations, it is clear that the momentum balance evaluation strictly depends on local channel bed slope. However, when dealing with mobile bed systems, where the channel conformation can rapidly change, the estimation of bottom slope may be difficult and the use of approximations can lead to undesirable numerical errors. This aspect becomes particularly evident in the presence of fully developed bed forms and in narrow channels presenting abrupt geometry discontinuities, caused, for example, by the collapse of transversal structures such as check dams or bottom steps. This paper, therefore, focuses on this aspect and proposes a mathematical reformulation of the balance equations. The advantages of this approach have been illustrated in the case of one-dimensional (1D) streams, and have already been extended to 2D applications in Valiani and Begnudelli (2006). The MacCormack (1969) explicit finite difference scheme has been chosen for numerical implementation, since it has a very simple structure and is able to reveal all the advantages of the proposed approach. The presented formulation has already been demonstrated to improve numerical stability relating to fixed bed test cases, indicating it to be particularly suitable also for morphological evolution studies (Schippa and Pavan, in press). The basic idea is to consider the pressure term due to the non-cylindrical portion of the channel as a function of static moment for the cross-section variation computed at a constant water level. In this way, the governing equations are completely independent of any arbitrary hypothesis about the longitudinal bed slope and channel geometry, thus they are appropriate for natural rivers and complex channel configurations.

A set of laboratory test cases have been used to test the potentialities of the model. Each test case was chosen to represent a particular problem which has often been encountered in natural rivers. In this way, the model has been tested on different flow regimes, including transition through critical conditions and evolution mechanisms, such as erosion, deposition and knickpoints migration.

To better highlight the advantages deriving from the proposed scheme, the results of the simulations have been compared to those obtained with the standard formulation of balance equations, where the solution explicitly depends on the source term associated with the local bed slope. Numerical results are also compared with experimental data.

2. Governing equations

The proposed numerical model is based on a set of conservation laws written for channels of 1D flows with complex geometry (see Fig. 1). The equations express the mass conservation for the liquid phase (1), the momentum

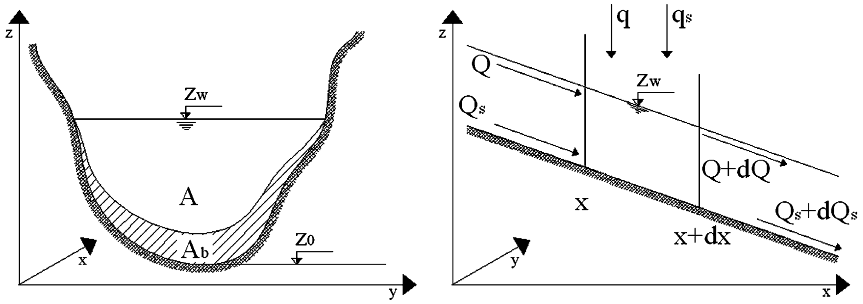

Fig. 1. Cross-section scheme and variables definition.