International Journal of Industrial Organization 45 (2016) 38–46

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Position auctions with dynamic resizing

Patrick Hummel

Google Inc., 1600 Amphitheatre Parkway, Mountain View, CA 94043, USA

A R T I C L E I N F O

Article history:
Received 20 May 2015
Received in revised form 24 November 2015
Accepted 4 December 2015
Available online 2 January 2016

JEL classifications:
D44
D47
D82

Keywords:
Position auctions
Dynamic resizing
Reserve prices
Mechanism design
Online advertising
Vickrey–Clarke–Groves

A B S T R A C T

This paper analyzes mechanisms for selling advertising inventory in a position auction in which displaying less than the maximal number of ads means the ads that are shown can be dynamically resized and displayed more prominently. I characterize the optimal mechanism with and without dynamic resizing, and illustrate how the optimal reserve prices in a Vickrey–Clarke–Groves mechanism vary with the amount of dynamic resizing and the number of bidders.

1. Introduction

In standard Internet auctions where multiple positions for advertising opportunities are being auctioned at the same time, it is typical for the size and the prominence of the ads that are displayed to be independent of the number of ads that are shown. This is the case in standard sponsored search auctions where a fixed amount of space is allocated for each ad on the side of the search page regardless of the number of ads that end up being displayed.

But while it is standard for the size and the prominence of the ads that are displayed to be fixed in most standard Internet position auctions, this is not the case for all such auctions. The Google Display Network (GDN) helps independent publishers monetize their websites by finding appropriate advertisers for their websites and then running an auction to select ads that should be displayed next to the content on a publisher's website. Many of these auctions are for multiple advertising positions on the website, and have the feature that displaying fewer ads will enable the ads that are displayed to be dynamically resized and shown more prominently, and thus receive a larger number of clicks than if more ads had been displayed on the site. In such a setting, if the most valuable ads are significantly more valuable than some of the less valuable ads that could be displayed, one may prefer to show a smaller number of the most valuable ads so that these ads will receive more clicks.

While position auctions with dynamic resizing are used to auction off a wide amount of advertising inventory on GDN, to the best of my knowledge there has been no published work that theoretically analyzes the properties of these auctions. This paper fills this gap in the literature by analyzing the properties of mechanisms that could be used by Google to sell content ads on GDN. Much of the paper analyzes the properties of the Vickrey–Clarke–Groves (VCG) mechanism that is currently used on GDN.

I first characterize properties of the optimal reserve prices in a VCG mechanism. In a standard position auction without dynamic resizing, the optimal reserve price is the same as the optimal reserve price in a standard Vickrey auction for a single object. However, under dynamic resizing, the optimal reserve price for the publisher is greater than this.

Next, I address the question of whether this VCG mechanism with reserve prices is optimal. In a standard position auction without dynamic resizing, I show that there is no feasible mechanism that will lead to greater revenue for the publisher than running a VCG mechanism with the optimally chosen reserve price. But with dynamic resizing of ads, the optimal mechanism will instead be a direct revelation mechanism that maximizes efficiency with respect to the virtual valuations rather than the actual valuations. This optimal mechanism will typically display fewer ads than the VCG mechanism with reserve prices.

Finally, I give some comparative statics results for a special case of the model in which no more than two ads may be displayed. In this setting, I illustrate that the optimal reserve price in the VCG mechanism

E-mail address: phummel@google.com.

http://dx.doi.org/10.1016/j.ijindorg.2015.12.002
0167-7187/

will be increasing in the number of bidders and vary non-monotonically with the amount of dynamic resizing that takes place.

Position auctions with dynamic resizing are a type of auction with variable supply. Auctions with variable supply have been analyzed in the literature (*e.g.* Back and Zender, 2001; Damianov and Becker, 2010; Damianov et al., 2010; Lengwiler, 1999; LiCalzi and Pavan, 2005), but none of these papers can be applied to the Internet position auction setting since these papers focus on models with perfectly divisible goods, and they also explicitly restrict attention to discriminatory-price and uniform-price auctions, neither of which are used in Internet position auctions. Perhaps the most closely related paper of these is Ausubel and Cramton (2004), which considers the possibility of Vickrey auctions with reserve prices in an auction setting with variable supply. However, since this paper again focuses on perfectly divisible goods, it cannot be applied to the Internet position auction setting.

The fact that ads will be less likely to receive clicks if displayed alongside other ads is also a form of an allocative externality. Auctions with allocative externalities have been studied by Katz and Shapiro (1985, 1986) in the context of auctioning licenses to operate a technology or use a patent. Jehiel and Moldovanu (1996) and Jehiel et al. (1996, 1999) have also studied models with externalities in which the losing bidders' payoffs may be affected by the identity of the winning bidder. And Chen and Potipiti (2010) studies a setting with externalities in which a losing bidder's payoff may be decreasing in that bidder's type. These papers all differ from my paper in that they focus on auctions for a single object.

More closely related to the present paper, Jehiel and Moldovanu (2001) and Figueroa and Skreta (2011) have both considered auctions with externalities in which the auctioneer may sell multiple different objects in the same auction. Jehiel and Moldovanu (2001) derive necessary conditions for an efficient and incentive compatible mechanism to exist, and Figueroa and Skreta (2011) illustrate a number of insights about optimal mechanisms, including that revenue-maximizing reserve prices may depend on the bids of other buyers, and revenue-maximizing mechanisms may sell too often. However, these papers do not attempt to derive results that are specific to a position auction with dynamic resizing of ads.

Finally, there has been a variety of work analyzing mechanisms for selling multiple advertising opportunities for various positions at the same time. Some of these papers, such as Aggarwal et al. (2008); Athey and Ellison (2011); Chen and He (2011); Giotis and Karlin (2008); Gomes et al. (2009); Kempe and Mahdian (2008), and Kuminov and Tennenholtz (2009) explicitly model consumer search which may result in externalities arising from ad click-through rates being influenced by which other ads are shown on the page. Fotakis et al. (2011); Ghosh and Mahdian (2008), and Hummel and McAfee (2014) present alternative models of position auctions with externalities in which the click-through rates of an ad may be influenced by which other ads are on the page.[1] However, this literature has not considered scenarios in which displaying less than the maximal number of ads may result in dynamic resizing of ads which causes the ads that are displayed to receive a greater number of clicks. Furthermore, most papers on position auctions (*e.g.* Aggarwal et al., 2006; Börgers et al., 2013; Chen et al., 2009; Edelman et al., 2007; Edelman and Schwarz, 2010; Fukuda et al., 2013; Gomes and Sweeney, 2014; Lahaie, 2006; Varian, 2007, and Yenmez, 2014) do not allow for the possibility that the click-through rates of ads may be influenced by how many other ads appear on the page.

The paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the properties of the optimal reserve prices for the VCG mechanism and the optimal mechanism and compares the properties of VCG with reserve prices to the optimal mechanism. Section 4 presents some comparative statics results for the case where a maximum of two ads may be displayed. Finally, Section 5 presents some simulation results comparing different mechanisms and Section 6 concludes.

[1] Also see Burguet et al. (2015) and White (2013) for work on the interaction between organic search results and advertisements.

2. The model

There are n advertisers in the set $N=\{1,\dots,n\}$, and each advertiser i has a value v_i for a click on one of the advertiser's advertisements that is an independent draw from the cumulative distribution function $F_i(\cdot)$ with corresponding continuous density $f_i(\cdot)$. Each advertiser i simultaneously submits a bid $b_i \geq 0$ for the right to place an ad on a publisher. After receiving these bids, a system decides the order in which the ads should be displayed on the publisher's site and possibly also decides how many ads should be displayed. Throughout I assume that no more than $s \leq n$ ads can be displayed on the publisher's site.

The number of clicks that an advertiser receives will be affected both by the total number of ads displayed and the order in which these ads are displayed. In particular, if a total of k ads are displayed on the publisher's site, then the ad in the jth-highest position receives a total of $x_{j,k}$ clicks, where $x_{j,k}$ is nonincreasing in j for all k and $x_{j,k}=0$ for all $j>k$. The total payoff to an advertiser i who receives x clicks is then $x(v_i-c_i)$, where c_i represents the cost an advertiser must pay per click.

Throughout the paper I consider situations in which the publisher only shows an advertiser's ad on a page if the advertiser pays a certain minimum reserve price r for each click. In order to achieve this, I focus on a mechanism that I refer to as *VCG with reserve prices*. This mechanism will ensure that all advertisers have an incentive to bid truthfully and that any advertiser who has an ad displayed is required to pay a price of at least r per click.

Informally, this mechanism proceeds by considering all K bidders who submit a bid greater than the reserve price and restricting attention to allocations with no more than K ads. Prices are then set using VCG pricing under the assumption that there is an additional bidder who places a bid equal to the reserve price. In this situation, any advertiser who has an ad shown necessarily pays a price per click that is greater than or equal to the reserve. Similarly, since the VCG mechanism is a truthful mechanism, it follows that the advertisers have an incentive to bid truthfully in this framework.[2]

Formally, let a denote the number of advertisers who submitted a bid per click that is greater than or equal to the reserve price r, and let $K\equiv\min\{a,s\}$ be the smaller of the number of slots or the number of advertisers with a bid greater than or equal to the reserve. For $j\leq K$, define $b_{(j)}$ to be the jth-highest bid submitted by any of the advertisers. And for $j=K+1$, define $b_{(j)}$ to be the larger of the reserve price and the jth-highest bid.

The VCG mechanism with reserve price r proceeds as follows: The mechanism displays a total of k ads, where k is the positive integer in [1, K] that has the highest value of $\sum_{j=1}^{k} x_{j,k} b_{(j)}$, by displaying the ad with the jth-highest bid in the jth-highest of these positions. That is, the mechanism selects the allocation that would maximize efficiency subject to the constraint that we can only show ads from advertisers who bid more than the reserve.

Then if $S_{j,K}\equiv\max_{k\in[1,K]}\sum_{i=1}^{j-1} x_{i,k} b_{(i)}+\sum_{i=j}^{k} x_{i,k} b_{(i+1)}$ and $R_{j,k}\equiv\sum_{i\neq j} x_{i,k} b_{(i)}$, the advertiser in the jth position is charged a total price per click of $\frac{1}{x_{j,k}}\left(S_{j,K}-R_{j,k}\right)$. Here $S_{j,K}$ is a term representing the total welfare that could be achieved if the bidder with the jth-highest bid were not in the auction when there is an additional bidder who submits a bid of r and no more than K ads can be displayed. $R_{j,k}$ represents the total welfare of all advertisers except for the advertiser with the jth-highest bid. These prices are thus identical to the prices that would be selected under VCG if no more than K ads could be displayed and

[2] Another possible way to implement reserve prices in the VCG mechanism would be to increase bidder payments *ex post* if they are insufficient to meet the reserve. However, such an implementation might not preserve incentive compatibility for advertisers even if there is no dynamic resizing (Even-Dar et al., 2008).