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Programs for kriging and sequential Gaussian simulation with locally varying anisotropy using non-Euclidean distances ☆

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ABSTRACT

Geological deposits display nonlinear features such as veins, channels or folds that result in complex spatial anisotropies that are difficult to model with currently available geostatistical techniques. The methodology presented in this paper for incorporating locally varying anisotropy in kriging or sequential Gaussian simulation is based on modifying how locations in space are related. Normally, the straight line path is used; however, when nonlinear features exist the appropriate path between locations follows along the features. The Dijkstra algorithm is used to determine the shortest path/distance between locations and a conventional covariance or variogram function is used. This nonlinear path is a non-Euclidean distance metric and positive definiteness of the resulting kriging system of equations is not guaranteed. Multidimensional scaling (landmark isometric mapping) is used to ensure positive definiteness. In addition to the variogram, the only parameters required for the implementation of kriging or sequential Gaussian simulation with locally varying anisotropy are (1) the local orientation and magnitude of anisotropy and (2) the number of dimensions required for multidimensional scaling. This paper presents a suite of programs that can be used to krige or simulate practically sized geostatistical models with locally varying anisotropy. The programs kt3d_LVA, SGS_LVA and gamv_LVA are provided.

1. Introduction

Anisotropy refers to different continuity of a deposit in different directions. If the direction and magnitude of anisotropy are well understood, they can be incorporated to increase the accuracy of modeling. Eriksson and Siska (2000) present a description of anisotropy and its application to traditional geostatistical algorithms that assume second order stationarity, such as kriging and sequential Gaussian simulation (SGS).

Stationarity is the decision of how to pool the available data for analysis. Assuming the modeling domain is first order stationary implies that the mean of a random function, $Z(\mathbf{u})$, within a domain, A, is constant at all locations, $\mathbf{u}$ (Eq. (1)). This allows data to be pooled to calculate the global mean of a deposit. In cases where the mean varies across the deposit the assumption of first order stationarity no longer holds. McLennan (2008) presents a detailed review of current techniques for modeling deposits that violate first order stationary:

$$E\{Z(\mathbf{u})\} = m \quad \forall \mathbf{u} \in A \tag{1}$$

Second order stationary assumes that the covariance function of a random variable is invariant under translation, i.e. $C(\mathbf{h})$ is constant within the modeling domain. The covariance between any two points separated by a lag distance, $\mathbf{h}$, is given as

$$C(\mathbf{h}) = E\{Z(\mathbf{u}+\mathbf{h})\cdot Z(\mathbf{u})\} - m^2 \quad \forall \mathbf{u} \in \mathbf{A} \tag{2}$$

where $\mathbf{h}$ is a lag vector between two locations in space. The covariance at lag $\mathbf{h}=0$ is the variance. The focus in this paper is on the spatial features, $\mathbf{h} > 0$. There are few techniques for considering domains that are not second order stationary and they have received less attention in the literature than first order stationarity.

Often, deposits display the type of anisotropy shown in Fig. 1 and clearly violate the assumption of second order stationarity. Techniques such as kriging provide disappointing results because only a single direction of continuity can be incorporated (Fig. 2 left). More geologically realistic results are obtained by considering locally varying anisotropy (LVA) (Fig. 2 right); however, incorporating LVA is difficult with available techniques. Often simple anisotropies, as in Fig. 2, can be accounted for with implementation details such as stratigraphic gridding or domaining, but more complex anisotropy fields (Fig. 1 left) remain a challenge. Further motivation for considering LVA in geostatistical modeling comes from highly continuous but locally varying deposits such as fluvial reservoirs (Fig. 3).

Estimates of resource volume and local predictions in these situations are improved by considering LVA. The methodology

☆ Code available from server at http://www.ualberta.ca/~jbb/LVA_code.html or at http://www.iamg.org/CGEditor/index.htm.

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Fig. 1. Cross sections displaying LVA. Left: Folding caused by San Andreas Fault (Kessell, 2010), plot covers approximately 100 m. Right: Folding in northern Rocky Mountains (Kutis, 2007), plot covers approximately 300 m. After Boisvert and Deutsch (2009).

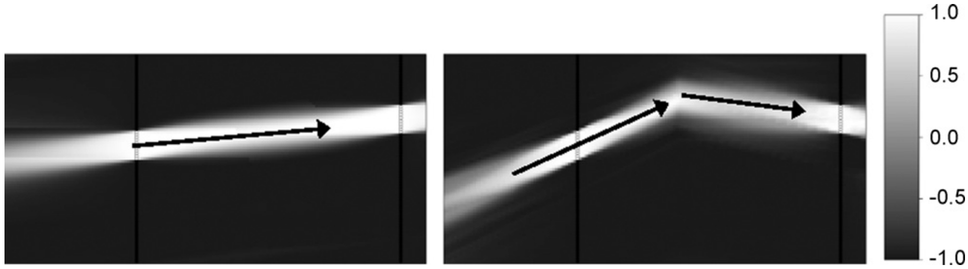

Fig. 2. Modeling a coal seam (Fig. 1) with two drill holes. Left: Traditional kriging. Right: Kriging considering LVA. Plot dimensions are nominally 300 m × 150 m with grade in Gaussian units.

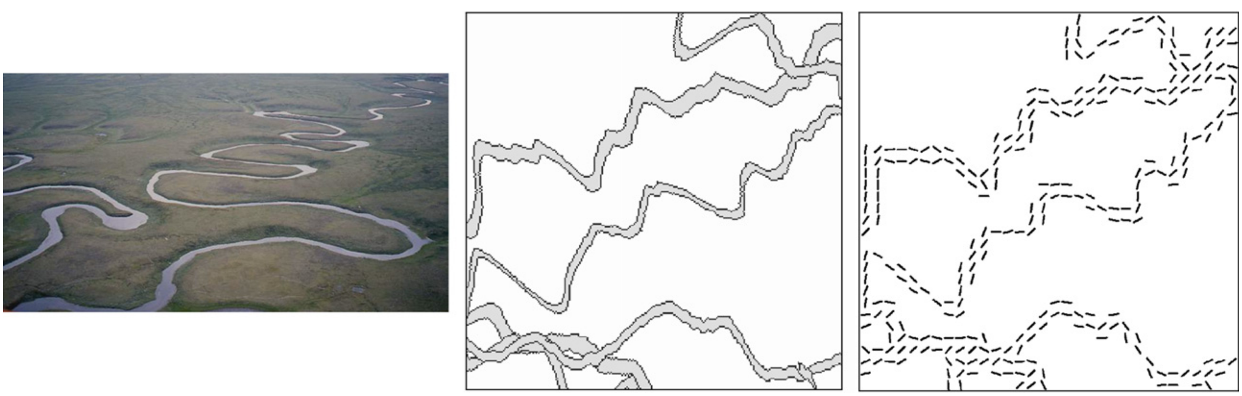

Fig. 3. Left: A meandering channel (Ingolfsson, 1997). Middle: A conceptual model of channels in a fluvial reservoir (Pyrcz et al., 2008). Right: A map of locally varying directions.

presented in this paper allows for estimation or simulation within a domain where second order stationarity does not hold.

2. Previous work

Much of the past work integrating LVA into geostatistical modeling implements kriging with a local search (Deutsch and Lewis, 1992; Sullivan et al., 2007; Xu, 1996). To determine the kriging weights the anisotropy specification at the estimation location is applied to its local neighborhood; Fig. 4 shows an exaggerated example where the anisotropy directions at the estimation locations are drastically different, but it highlights the limitations of incorporating LVA in this way. This idea has been extended to spectral methods where the spectral functions are considered locally variable (Fuentes, 2002a, b) but it is still assumed that within an arbitrary region these spectral functions are stationary.

Stroet and Snepvangers (2005) have recently proposed a variant of kriging where LVA is automatically calculated from the available data using an iterative image analysis technique. This technique requires sufficient data density to directly infer the underlying LVA. Often in petroleum and mining applications, the data does not clearly reveal the curvilinear features due to large sample spacing but these features are known to exist based on additional information. In these cases, this iterative image analysis technique could not be applied.

Environmental modeling must often consider local anisotropies because of the nature of the variables considered. Pollutant spread, rain fall patterns, animal migration, etc. exhibit nonlinear features and display LVA in space or time. The pioneering work of Sampson and Guttorp (1992), which has been expanded upon by multiple authors, led the way for much of the work on LVA in an environmental context. They utilize multidimensional scaling (MDS) to incorporate LVA into modeling. Their approach is limited to smaller models as they use classical MDS (Mardia et al., 1979). Moreover, they assume that there are repeated measurements at individual