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Secret reserve prices in first-price auctions

Frank Rosar *

Department of Economics, University of Bonn, Lennéstr. 37, 53113 Bonn, Germany

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ABSTRACT

This article offers a theoretical explanation for the use of secret reserve prices in auctions. I study first-price auctions with and without secret reserve price in an independent private values environment with risk-neutral buyers and a seller who cares at least minimally about risk. The seller can fix the auction rules either before or after she learns her reservation value. Fixing the rules early and keeping the right to set a secret reserve price can be strictly optimal. Moreover, I describe the relation of using a secret reserve price to phantom bidding and non-commitment to sell.

1. Introduction

While reserve prices are often kept secret in practice (e.g., Elyakime et al., 1994; Ashenfelter, 1989), non-standard assumptions are needed to justify their use on a theoretical basis. In the symmetric independent private values auction environment with a regular distribution, risk-neutral buyers and a risk-neutral seller, the optimal mechanism is implemented by any standard auction with an optimally chosen announced reserve price (Myerson, 1981). Secret reserve prices may be used to increase participation in second-price auctions with common values (Vincent, 1995), to credibly signal information in repeated second-price auctions (Horstmann and LaCasse, 1997), to induce more aggressive bidding in first-price auctions with risk-averse bidders (Li and Tan, 2000), and in first-price and second-price auctions with reference-based utility (Rosenkranz and Schmitz, 2007). I offer a further theoretical explanation for why secret reserve prices might be used in first-price auctions. My explanation is based on seller information that improves over time and risk-aversion on the seller's side.

In practice, a seller often fixes and announces the rules of an auction some time before the auction does actually take place. While this is sometimes necessary for exogenous reasons (e.g., because potential buyers need to prepare bids), the seller has normally at least the possibility to announce the rules of the auction some time in advance. During such a time, the seller's information might improve. For example, she might get better informed about her own use value or a new outside option might arise. I explain in this article why there can be a role for using a secret reserve price in a first-price auction when either (1) the seller's information improves for exogenous reasons after she fixes the rules of the auction and before the auction is conducted or (2) the seller is risk-averse and she can endogenously induce a situation in which this happens.

I proceed in two steps. I first analyze in Section 2 the case in which the seller's information improves for exogenous reasons. While I stick to the independent private values model with risk-neutral buyers and a risk-neutral seller, I consider a timing in which the seller has to commit to the rules of a first-price auction before she learns her value. The seller chooses the bid space and whether she keeps the right to set a secret reserve price later on when she is informed. A secret reserve price might be part of the optimal auction rules. The result arises quite naturally in this setting at the cost that it relies on a timing which might seem artificial. Then I show in Section 3 that the (artificial) timing of Section 2 can arise endogenously. If the seller cares about risk, she does under certain conditions prefer to commit to the auction rules early before she is informed to waiting until she is informed and fixing the rules then. By committing to the auction rules early, the seller induces a bidding behavior which does not vary in her own value. In conjunction with a secret reserve price, it can be possible to use this as an instrument to make the induced profit distribution less risky without sacrificing (too much) expected profit.

* Tel.: +49 228 73 6192; fax: +49 228 73 7940.
E-mail address: email@frankrosar.de.

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In the main parts of this article, I employ the simplest model that allows me to demonstrate the effects which drive my results. It relies on a binary seller value and—in the second part of the article—on seller preferences which are lexicographic in expected profit and variance of profit. The assumption of a binary seller value is mainly for technical convenience. The analysis becomes more complicated and the results become less clean when the seller's value is continuously distributed, but the crucial effects extend also to continuous distributions (see Subsection 2.4). The assumption of lexicographic preferences works against my effects. It simplifies however the analysis and, more importantly, it makes analysis and results better comparable to standard auction theory (see Subsection 3.3).

2. Exogenous seller information

2.1. The model

There is a seller of an indivisible object and two potential buyers, buyer 1 and buyer 2. I denote a generic buyer by i and the other buyer by $-i$. The values that the seller and the buyers attribute to the object are realizations of the independently distributed random variables X_0, X_1 and X_2. Let $X := (X_0, X_1, X_2)$. I use lower case letters to denote realizations of these random variables. X_i is distributed according to a cumulative distribution function F with support [0, 1], a continuous and strictly positive density function f and a strictly increasing function $J(x_i) := x_i - (1 - F(x_i))/f(x_i)$. $J(x_i)$ describes the virtual valuation function introduced by Myerson (1981) which is important for many auction theory problems. X_0 is 0 with probability $p \in (0, 1)$ and $z \in (0, 1)$ otherwise. When I denote the indicator variable which describes whether buyer i obtains the object by $q_i \in \{0, 1\}$ and buyer i's payment to the seller by t_i, then buyer i's profit is given by $q_i x_i - t_i$ and the seller's profit is given by $\pi = (1 - q_1 - q_2)x_0 + t_1 + t_2$.

I am interested in first-price sealed-bid auctions with and without a secret reserve price.[1] The timing is as follows: First, the seller chooses a closed bid space $B \subset \mathbb{R}_+$ and whether she will set a secret reserve price in Stage 3 ($S = y$) or not ($S = n$). I will denote the lowest admissible bid by r_a and refer to it as announced or open reserve price. The auction rules B and S are observable to the buyers. Second, each player privately learns his value. Third, if $S = y$, the seller chooses a secret reserve price $r_s \in \mathbb{R}_+$. Moreover, each buyer i either submits a sealed bid $b_i \in B$ or does not participate in the auction. I denote non-participation with $b_i = \varnothing$ such that a strategy of a buyer is described by a function $b : [0, 1] \rightarrow B \cup \{\varnothing\}$. If $S = n$ (resp. $S = y$), the seller keeps the object when no bid (resp. no bid $b_i \geq r_s$) is submitted. Otherwise, the buyer with the highest bid obtains the object and pays his bid to the seller. Ties between the buyers are broken according to a fair lottery.

Each buyer is risk-neutral such that he strives for maximizing his expected profit. I consider at first the case in which the seller is risk-neutral as well. Later I will discuss the case in which she has lexicographic preferences in expected profit and variance of profit and the case in which she is risk-averse with non-lexicographic preferences. I am interested in undominated Perfect Bayesian Equilibria where participation and bidding behavior is symmetric across buyers (usPBE).[2]

To simplify the exposition of my results, I assume that a buyer participates whenever he is indifferent between participation and non-participation and that he chooses the higher bid whenever he is indifferent between a higher and a lower bid. Moreover, for any $b_i \in \mathbb{R}_+$ I write $b_i > b_{-i}$ to describe the case in which either $b_{-i} = \varnothing$ or $b_{-i} \in [0, b_i)$.

[1] Why the seller uses a first-price payment rule lies outside of my model. One reason might be that first-price auctions perform well when the seller is risk-averse which is the case in which I will be finally interested in (Waehrer et al., 1998).

[2] If the seller sets a secret reserve price, it has a second-price character for her. To exclude equilibria in which the seller sets a secret reserve price which is prohibitively high and in which the buyers submit no bids, I restrict attention to PBE which do not rely on weakly dominated strategies.

2.2. Strategic bidding behavior and the effect of holes in the bid space

My model generalizes a standard independent private values first-price auction model in three respects: First, the seller's information improves over time. She is better informed at the time the auction is conducted than at the time she designs and announces the auction rules. Second, I explicitly allow the seller to restrict the set of admissible bids further than by setting only an open reserve price. Third, I allow for the possibility that the seller sets a secret reserve price before the auction starts. I explain in this subsection the equilibrium behavior in the subgame which is played after the auction rules S and B are fixed and I describe how the three generalizations affect the analysis.

Consider first how the seller is affected by the secret reserve price decision S for a given behavior of the buyers. If $S = n$, it does not depend on the realization of the seller's value x_0 whether the object is sold or not. The object is sold at the highest bid whenever at least one bid is submitted. If the object is not sold, the seller keeps the object and realizes an expected profit of $\mathbf{E}_X[X_0] =: \bar{x}_0$. By contrast, if $S = y$, the selling decision can depend through the secret reserve price on the seller's private information x_0. She sells the object if the highest bid exceeds the secret reserve-price and keeps it otherwise. Intuitively, the highest bid can be interpreted as a take-it-or-leave-it offer to buy the object and the secret reserve price describes the threshold above which these offers are accepted by the seller. As the value of the secret-reserve price does not feed back on the buyers' bidding behavior, setting $r_s = x_0$ is clearly optimal for the seller. The seller's profit is thus the maximum of the highest bid and her value x_0 if at least one bid is submitted and she realizes an expected profit of $\bar{x}_0$ otherwise. Hence, $S = y$ implies that the seller faces a commitment problem regarding under which conditions she will sell the object, whereas $S = n$ implies that the selling decision is not affected by the seller's value x_0.[3]

The difference in the selling behavior for $S = n$ and $S = y$ may induce for the same bid space B a different behavior by the buyers. Consider thus the problem a buyer i faces when he has value x_i and believes that the other buyer behaves according to a strategy $b : [0, 1] \rightarrow B \cup \{\varnothing\}$. If $S = n$, buyer i faces the problem either not to participate or to choose a bid $b_i \in B$ to maximize

$$\left[\text{Prob}_X\{b_i > b(X_{-i})\} + \frac{1}{2}\text{Prob}_X\{b_i = b(X_{-i})\}\right] \times (x_i - b_i). \quad (1)$$

He faces a trade-off between a higher probability of winning against the other buyer and a higher rent conditional on winning. If $S = y$, there is an additional effect. A higher bid may then also induce a higher probability with which the seller does actually sell the object besides increasing the probability of having the highest bid. Buyer i's expected profit from submitting a bid $b_i \in B$ is then

$$\left[\text{Prob}_X\{b_i > b(X_{-i})\} + \frac{1}{2}\text{Prob}_X\{b_i = b(X_{-i})\}\right] \times \text{Prob}_X\{b_i \geq X_0\} \times (x_i - b_i). \quad (2)$$

The following lemma summarizes equilibrium properties which follow from standard reasoning and which hold equally for $S = n$ and for $S = y$:

Lemma 1. *Fix any S and any B with $r_a < 1$. If $b : [0, 1] \rightarrow B \cup \{\varnothing\}$ is part of a symmetric equilibrium of the game which is played after B and S are chosen, then the following is true:*

- **(a)** *There is threshold participation behavior and the participation threshold corresponds to the lowest admissible bid r_a.*
- **(b)** *The buyers' bidding behavior $b(x_i)$ is weakly increasing on $[r_a, 1]$ with $b(r_a) = r_a$.*

[3] Setting a secret reserve price is closely related to placing a phantom bid and to not committing to sell after observing the bids. See Section 4 for a discussion.