Multi-dimensional price discrimination ☆

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ARTICLE INFO

ABSTRACT

We examine the profitability and welfare implications of price discrimination in a multi-dimensional model. First, when firms price discriminate on one and the same dimension, uniform price lies in between discriminatory prices and price discrimination raises profits relative to uniform pricing. This is in contrast to common findings in existing one-dimensional models featuring best-response asymmetry, suggesting that price discrimination can have qualitatively different implications in one- and multi-dimensional models. Second, price discrimination on one and the same dimension is the likely outcome when price discrimination decisions are endogenized using a two-stage discrimination-then-pricing game. Correspondingly, an observation of one-dimensional price discrimination in practice does not necessarily indicate that the underlying model should be one-dimensional.

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1. Introduction

Consumer characteristics usually are multi-dimensional (e.g., location, gender, age, income and student status). Correspondingly, if endowed with information on such consumer characteristics, a firm may be able to price discriminate on multi-dimensions.[1] In this paper, we examine the welfare implications of third-degree price discrimination in a multi-dimensional model. We ask two questions: First, how should price discrimination be conducted? Second, what are its welfare implications?

A relatively large literature has answered these questions in one-dimensional settings where consumer heterogeneity occurs on a single dimension along which firms can price discriminate (see Armstrong, 2006; Stole, 2007 for surveys of this literature). One strand of this literature assumes best-response symmetry (e.g., Holmes, 1989) and common findings are that uniform price lies in between discriminatory prices and price discrimination may raise or lower profits.[2] Another strand assumes best-response asymmetry and usually finds that price discrimination intensifies competition, benefiting consumers at the cost of firms (e.g., Bester and Petrakis, 1996; Chen, 1997; Shaffer and Zhang, 1995; Thisse and Vives, 1988).[3] This paper extends the existing analysis to a multi-dimensional setting and several new questions emerge. Would firms have an incentive to price discriminate on some dimensions but not others? And if they do, would they price discriminate on the same or different dimensions? These questions do not fit existing studies, because their underlying one-dimensional

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[1] For example, movie theaters charge different prices for students vs. non-students. Membership dues for many professional associations (e.g., AEA) depend on member's income level. Nightclubs charge different prices for males vs. females.

[2] Robinson (1933) characterizes the two markets served by a monopolist as "strong" and "weak" markets in the sense that its discriminatory price is higher (lower) in the strong (weak) market. This characterization has been extended to imperfectly competitive markets, and best-response symmetry is defined as occurring when firms' strong markets coincide. In contrast, under best-response asymmetry, one firm's strong market is the other firm's weak market (Corts, 1998).

[3] Corts (1998) shows that best-response asymmetry is necessary but not sufficient for price discrimination to lead to all-out competition. However, it is commonly shown, especially in a location type of models such as Hotelling model (e.g. Armstrong, 2006), that price discrimination based on a consumer characteristic featuring best-response asymmetry is a prisoners' dilemma game. On the other hand, if price discrimination is based on a characteristic featuring best-response symmetry (e.g., 'choosiness' in Armstrong, 2006), then price discrimination can improve profits.

models do not give firms the option of price discriminating on some dimensions but not others.[4] Moreover, as we will show later, even when product differentiation occurs on multi-dimensions, firms may still choose to price discriminate on only one dimension. Correspondingly, an observation of one-dimensional price discrimination is not necessarily a confirmation that the underlying model is one-dimensional.

To obtain concrete results, we employ a model commonly used in the existing literature featuring best-response asymmetry (e.g., Liu and Serfes, 2004; Shaffer and Zhang, 1995; Thisse and Vives, 1988). In particular, we modify the standard one-dimensional Hotelling model to two-dimensions in our main analysis and to general n-dimensions in the extensions. Similar to much of the literature, we assume that price discrimination along a given dimension takes the form of two-group price discrimination.[5] Our results are qualitatively different from those in one-dimensional models. For example, we find that when firms price discriminate on one and the same dimension, profits go up and uniform price lies in between the discriminatory prices. This is in sharp contrast to the common findings in one-dimensional models featuring best-response asymmetry. On the other hand, price discrimination on one but different dimensions and price discrimination on both dimensions intensify competition and reduce profits, similar to the results in one-dimensional models. While these exact results are specific to our model, their general message is clear: the welfare implications of price discrimination can be qualitatively different in one- and multi-dimensional models. Correspondingly, results based on a one-dimensional model can be inaccurate when the underlying model is multi-dimensional. Our study takes a step in examining this issue and calls for more research.

We identify two effects of price discrimination in our multi-dimensional model. First, the well-understood *intensified competition effect* exists in both one- and multi-dimensional models. That is, the ability to price discriminate enables a firm to be more aggressive in its weak market. However, when both firms do so, they force their rivals to be more aggressive in their strong markets as well, leading to all-out competition (all discriminatory prices are below the uniform price). Second, price discrimination has a *reduced demand elasticity effect* which exists in multi-dimensional models but not in one-dimensional models. This effect reduces competition and raises prices. Our results suggest that the reduced demand elasticity effect dominates the intensified competition effect when firms price discriminate on one and the same dimension, but the results are reversed when firms price discriminate on one but different dimensions or when they price discriminate on both dimensions.

We then endogenize firms' decisions on price discrimination using the framework that firms acquire information about consumers which enables them to price discriminate. Our results show that price discrimination on one and the same dimension can emerge as a SPNE if consumer information is not too costly to acquire. Price discrimination on both dimensions can also be supported as an SPNE. However, it is dominated by price discrimination on one and the same dimension from firms' perspective. Taken together, they suggest that if firms price discriminate in equilibrium, the likely outcome is price discrimination on one and the same dimension which lowers consumer surplus. This is contrary to the results in existing studies where price discrimination is characterized as a prisoners' dilemma game. Moreover, it suggests that an observation of one-dimensional price discrimination in practice does not necessarily imply that the underlying model is one-dimensional.

1.1. Related literature

One strand of the price discrimination literature analyzes the impacts of monopolistic price discrimination or oligopolistic price discrimination under best-response symmetry. In both cases, uniform price lies in between discriminatory prices in general.[6] The focus in the monopolistic price discrimination literature is usually on how price discrimination affects social welfare (e.g., Robinson, 1933; Schmalensee, 1981; Varian, 1985). Holmes (1989) extends the analysis to duopoly and examines the output and profit effects of price discrimination. He characterizes an elasticity-ratio condition in addition to the adjusted-concavity ratio in Schmalensee (1981). Armstrong (2006) considers price discrimination based on "choosiness" and finds that such price discrimination raises firms' profits.[7] In our model, when firms price discriminate on one and the same dimension, we obtain similar results despite the fact that our model exhibits best-response asymmetry.

Our paper is closely related to the literature on price discrimination with best-response asymmetry. See, for example, Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997) and Liu and Serfes (2004).[8] A recurring theme in this literature is that firms have an incentive to price discriminate but when both do so their profits go down relative to the uniform pricing level: a *prisoners' dilemma*. In our model, when firms price discriminate on one but different dimensions or price discriminate on both dimensions, we obtain similar results suggesting that price discrimination leads to (almost) all-out competition and lower profits.

While most studies find that price discrimination lowers profit under best-response asymmetry, there are exceptions. For example, Liu and Serfes (2013) show that perfect price discrimination can raise profit in two-sided markets. The key feature is that the cross-group externality drives the uniform pricing down but disappears when discriminatory prices hit a price floor of zero (negative prices are not allowed). In contrast, we consider multi-dimensional product differentiation without placing restrictions on marginal cost being low or requiring the existence of a price floor. Moreover, our qualitative results hold under both two-group (main model) and perfect price discrimination (extension), while the result in Liu and Serfes (2013) relies on the limit pricing nature of perfect price discrimination.

Our paper is also related to studies on multi-dimensional product differentiation. Tabuchi (1994) and Irmen and Thisse (1998) analyze firms' optimal location choice in multi-dimensional Hotelling models.[9] With consumers uniformly distributed on each dimension, they identify an equilibrium where firms maximize differentiation on one dimension, but minimizes differentiation on all other dimension(s) (max–min). Similar to these studies, we consider a multi-dimensional model. However, uniform pricing is assumed in all these studies while we allow price discrimination in this paper. Max–min product differentiation does not necessarily imply that one-dimensional model is adequate to analyze price discrimination. To see this, consider a simple example where firms' products differ in color only. Suppose that consumers' preferences for the two colors depend on gender (dimension 1) and student status (dimension 2). As long as these two dimensions are not perfectly correlated, one-dimensional model will not capture the full

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[4] The few exceptions include Lederer and Hurter (1986) and Anderson and De Palma (1988) who consider perfect price discrimination on all dimensions. In contrast, here we allow firms to price discriminate on only some of the dimensions. Note that the second-degree price discrimination literature has studied multi-dimensional screening (e.g., Armstrong and Rochet, 1999; Rochet and Stole, 2003).

[5] One can think of two-group price discrimination as being based on binary information regarding a consumer characteristic which the firms can use to divide consumers into two groups and price discriminate accordingly. Alternatively, one can think of it as a simplification of finer price discrimination, for example, perfect price discrimination, which is discussed in the Extensions.

[6] An exception is Nahata et al. (1990) who allow profit function to be not single-peaked and show that all discriminatory prices can go down.

[7] Armstrong and Vickers (2001) analyze duopoly price discrimination between two segments which differ in unit transport costs. They find that price discrimination raises profits if the unit transport costs are sufficiently small. Besanko et al. (2003) use aggregate data and show that targeted pricing need not generate prisoners' dilemma with more flexible empirical demand.

[8] These studies focus on one-stage games where firms charge higher prices to loyal customers. In contrast, Chen and Pearcy (2010) employ a dynamic game and find that firms may reward loyalty or entice brand-switching depending on the degree of preference dependence.

[9] Similar to these studies, our paper assumes horizontal differentiation on both dimensions. Other studies combine horizontal and vertical differentiation (e.g. Gans and King, 2006; Gilbert and Matutes, 1993).