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Efficient assignment mechanisms for liquidity-constrained agents

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ABSTRACT

We study alternative methods of assigning scarce resources to individuals who may be liquidity-constrained. Selling the resources via auctions is increasingly popular, but that method may produce an inefficient allocation when agents are liquidity constrained. A simple non-market scheme such as random assignment does better, if resale is allowed, since individuals with a high valuation but low liquidity are more likely to be assigned initially, and recipients with low valuations will resell to those with high valuations. Similarly, a need-based assignment scheme favoring those with low liquidity enhances welfare. Lotteries with entry fees could also be desirable. The optimal mechanism displays features of the non-market schemes such as in-kind and cash subsidies.

1. Introduction

Suppose that a government wishes to assign scarce resources to individuals or firms. Allocating resources to private citizens has been a major function of many governments, and its importance is growing with the increased use of privatization and outsourcing. The resources could be the rights to operate a business, to use public land or radio spectrum, to immigrate, to exploit natural resources, or to receive an education. Suppose that the objective of the government is to allocate the resources efficiently; i.e., assign them to those who will use them most productively.[1] Naturally, how best to perform that function has been a question that economists have grappled with for a long time.

The traditional answer to this question comes from the celebrated Coase theorem (Coase, 1960), which states that, given negligible transaction costs and well-defined property rights (which include the right to transfer ownership), it does not matter how a government assigns the goods initially. If the initial recipient is not the most efficient user of the good, a more efficient user will come along and offer to buy the good at a price that will benefit both, and this process will continue until the good is allocated efficiently. Given quasilinear utility for the parties, the eventual allocation of the good will be utilitarian efficient as the good will accrue to the individual who will use it most productively. This view, while a useful benchmark, provides little prescriptive guidance for answering policy questions. Also, the assumption of no transaction costs is unrealistic since the resale/bargaining process is often limited by various frictions. In that case, it is important to assign the resources as efficiently as possible in the first instance. How can one do this? The prevailing view among economists can be summarized by the "market principle": For the case of a homogeneous good, selling resources via competitive markets or via standard auctions such as uniform-price multi-unit auctions should do the job. Formally known as the first welfare theorem, the market principle is also quite intuitive: a competitive market selects for assignment those willing to pay the market-clearing price—but not those who are unwilling to pay it—so the recipients must value the goods more highly than do the non-recipients.

Despite its theoretical and intuitive appeal, many resources are assigned in ways that depart from the market principle. Seats in public schools, immigration visas, human organs, low-income housing and military duty are not assigned via competitive markets; rather, they are assigned at random or according to some priority rule, at

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[1] In practice, revenue may also be an important objective, which is justified on efficiency grounds as well, given the shadow cost of raising public funds.

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prices below the market clearing level, typically zero.[2] Non-market assignment methods have factored prominently in the assignment of public land. At the end of the 19th century, the U.S. government divided swaths of land in the American southwest into equal-sized lots and assigned them to settlers by a method famously known as the Oklahoma Land Rush; participants lined up at the starting line, raced to the lots of their choice, and staked claims to the ground.

Lotteries were also common for homesteading; participants registered by paying a nominal fee and were awarded a lot if their names were drawn. The use of lotteries ranges from modern-day Australia, 19th and early 20th century U.S. (as in Oklahoma and Georgia), as well as the biblical portrayal of the settlement of Jewish tribes in the Promised Land. Lotteries were also used to draft soldiers for the Vietnam War and to assign radio spectrum in the 1980s in the U.S. While the motivation and objectives behind the use of these methods vary (and may be non-economic), given their prevalence it is important to understand their efficiency implications. Can these methods be justified on efficiency grounds?

Non-market methods are unlikely to perform well in efficiency terms when no agents are liquidity-constrained, whereas competitive markets would work well, as intuition suggests. But we shall show that these non-market methods may have desirable efficiency implications when agents are liquidity-constrained, and they may even outperform competitive markets as a method of initial assignment. Liquidity constraints appear relevant in many instances, including those mentioned above, when the resources being allocated are valuable relative to one's cash holdings and they are difficult to collateralize. Then, those wishing to purchase the resource may not have the necessary cash or financing to pay the market price. Take the case of immigration. If a country were to sell immigration rights in an auction,[3] those who may develop skills valued highly in that country would be willing to pay more to win the rights, but they may be liquidity-constrained and unable to convince the capital market to fund the purchase. This story applies equally well to higher education as to the rights to operate a business or to use public land.

This reasoning suggests that the Coase theorem will not hold in the presence of agents' liquidity constraints precisely because the corrective function of resale will be limited by the constraints. More important, the market may not be the best method of *initial assignment* in that environment. Indeed, we show below that the market principle does not apply when agents' liquidity constraints are binding. Specifically, many nonstandard methods such as random assignment, lotteries with entry fees and need-based assignment (which favors agents who are cash-poor initially), can outperform competitive markets. We further show that features present in these mechanisms are part of the optimal mechanism we study later.

How alternative mechanisms perform in the presence of agents' liquidity constraints is a theme that has been studied in a growing literature, most of which concerns the revenue performance.[4] One exception is Maskin (2000), who studies the constrained-efficient mechanism with liquidity-constrained agents, given the constraint that the designer cannot provide transfers. The current paper will show how this latter constraint can be relaxed in a natural way and how doing so alters insights about the optimal mechanism. The current paper is adapted from our recent paper, Che et al. (2013), which develops the theme of the current paper in greater detail and generality. Some new themes and variations in the modeling approach emerge in the current paper, making it an accessible and yet comprehensive overview of the subject matter.

2. Model

Suppose that a designer has a mass $S < 1$ of an indivisible good to assign to a unit mass of risk-neutral agents. The canonical interpretation is that the designer is a government agency that wishes to assign multiple units of a homogeneous asset such as a license to operate a business or to exploit natural resources. For simplicity, we assume that there are only two goods: the indivisible good being allocated and a divisible numeraire, cash. Each agent realizes linear utility from cash, and demands one unit of the good being allocated. In the beginning, each agent is endowed with cash ("liquidity") of w and has a "valuation" v of the good, expressed in terms of the number of units of cash the agent is willing to pay.[5] The valuation can be interpreted as the amount of cash that the agent can generate by utilizing the good. The liquidity is the amount the agent has available to purchase the good, so he cannot pay more than w. We call the pair (w, v) an agent's type. Agents are risk-neutral with quasilinear preferences and face budget constraints equal to their liquidities. So, if a type-(w, v) agent receives the good with probability x and pays $t \le w$, he gets utility of $vx + w - t$.

An agent's liquidity is either high (w_H) or low (w_L), and the valuation is also either high (v_H) or low (v_L). Let $\pi_{ij} > 0$ denote the fraction of agents who are of type (w_i, v_j), for $i, j = L,H$, where $\sum_{i,j = L,H} \pi_{ij} = 1$. It is also useful to define the fraction $\phi := \pi_{LH} + \pi_{HH}$ of high-valuation agents and the fraction $\psi := \pi_{HL} + \pi_{HH}$ of high-liquidity agents. It is most interesting to focus on the situation in which the following assumption holds:

Assumption. (i) $0 \le w_L < v_L < v_H < w_H$ and (ii) $\pi_{HH} < S \le \psi$.

The first strict inequality in (i) allows us to focus on the interesting case in which the low-liquidity agents are constrained enough in their abilities to pay to produce an inefficiency when the resource is sold in the competitive market. Without this condition, a competitive market (more precisely, a standard auction with an appropriate rationing rule) will achieve full efficiency.[6] The last inequality in (i) ensures that high-liquidity agents are never constrained in the sense that they are always able to pay as much as they are willing to pay.

The first inequality in (ii) makes liquidity constraints relevant; without it, high-value high-liquidity agents will exhaust the entire supply of the good in a competitive market, so the liquidity constraint is never binding. The second inequality in (ii) is made for convenience; without this condition, the competitive market equilibrium is not well defined.[7]

Using the model, we shall compare competitive markets against the other methods of assignment mentioned above: random assignment, lotteries with entry fees, and need-based assignment. As we shall discuss, these other methods, although not as prominent as competitive markets, are used in various guises. Afterwards, we shall consider the full-fledged optimal (second-best efficient) mechanism.

[2] In the U.S., for example, random assignment and priority rules have both been used to assign seats in public schools, immigration visas and low-income housing to qualifying recipients. Priority rules are used for human organs whereas lotteries were used when there was a military draft.

[3] Becker (1987), Chiswick (1982) and Simon (1989) all discussed the idea of selling visas to qualified applicants.

[4] See Borgs et al. (2005), Che and Gale (1998, 2000, 2006), Condorelli (2012), Kotowski (2010), Hafalir et al. (2012), Laffont and Robert (1996), Maskin (2000), Pai and Vohra (2011), Pitchik (2009) and Richter (2011).

[5] More precisely, v is the amount the agent is willing to pay if he has an unlimited budget.

[6] If $w_L \ge v_L$, then the equilibrium price in the competitive market will be w_L in the case that $S \le \phi$ (so only the high-valuation consumers can be served), and v_L in case $S > \phi$. In the former case, all high-valuation consumers will demand the good, so they may need to be rationed, but the resulting outcome is efficient. In the latter case, the first-best is achieved since all high-valuation agents along with mass S-ϕ of low-valuation agents (who are indifferent to buying the good at that price) demand the good.

[7] The possible non-existence of a competitive equilibrium is due to both (i) indivisibility and (ii) budget constraints. Without(ii), Azevedo et al. (forthcoming) show that a competitive equilibrium exists. We show below the nature of non-existence in this case. See Footnote 8.