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Contest functions: Theoretical foundations and issues in estimation ☆

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ABSTRACT

Contest functions (alternatively, contest success functions) determine probabilities of winning and losing as a function of contestants' effort. They are used widely in many areas of economics that employ contest games, from tournaments and rent-seeking to conflict and sports. We first examine the theoretical foundations of contest functions and classify them into four types of derivation: stochastic, axiomatic, optimally-derived, and microfounded. The additive form (which includes the ratio or "Tullock" functional form) can be derived in all four different ways. We also explore issues in the econometric estimation of contest functions, including concerns with data, endogeneity, and model comparison.

1. Introduction

Contests are games in which each player exerts effort in order to increase his or her probability of winning a prize. There is a variety of areas of economics and other social sciences in which contests are applied. They include advertising by rival firms (Schmalensee, 1972, 1978), tournaments or influence-activities within organizations (Müller and Wärneryd, 2001; Rosen, 1986; Tsoulouhas et al., 2007), patent and other technology races (Baye and Hoppe, 2003; Reinganum, 1989), lobbying and rent-seeking (Nitzan, 1994; Tullock, 1980), litigation (Hirshleifer and Osborne, 2001; Robson and Skaperdas, 2008), wars and other types of conflict (Garfinkel and Skaperdas, 2007; Hirshleifer, 1995, 2000; Levitin and Hausken, 2010), political campaigns (Baron, 1994; Skaperdas and Grofman, 1995), as well as sports (Szymanski, 2003). Konrad (2009) provides an excellent introduction to the basic theory and applications of contests.[1]

How combinations of efforts by the players participating in a contest translate into probabilities of wins and losses is a critical component of a contest game. The functions that describe these probabilities as functions of efforts are often called *contest success* or simply *contest* functions.[2] In terms of their usage, they are analogous to production functions in production theory but they differ from production functions in two important ways. First, the outputs of contest functions are probabilities of wins and losses instead of deterministic outputs. Second, the inputs into contest functions, the efforts of the participating players, are *adversarially* combined so that a player's probability of winning is increasing in her or his effort but is decreasing in the efforts of all the adversaries.

The efforts themselves can be as varied as the particular social or economic environment to which the contest is meant to apply. In the case of tournaments and other intra-organizational competition the efforts are usually denominated in labor time expended. For advertising, lobbying, patent races, litigation, sports, wars, or political campaigns the cost of effort is typically represented by monetary

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[1] For an illuminating survey of contests and its various applications see Corchón (2007).

[2] We use the simpler second term in this paper even though one of the authors was one of the first users of the former term (Skaperdas, 1996), for reasons that are not clear to him at this time. Probably, he followed Hirshleifer (1989) who used the term "conflict and rent-seeking success functions" in his exploration of different functional forms.

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expenditures but the effort itself can be the output of an ordinary production function that is a function of a large number of inputs (purchased with money). For advertising, the efforts can be advertising messages that are produced by means of different types of specialized labor (artistic staff, creative staff, film directors and crews, and so on) and all the capital and other inputs that go together with them. For lobbying, the efforts can be varied from the face-time of lobbyists with political decision-makers to grass-roots organizing, produced by means of different types of labor, capital, and material inputs. For sports, although the direct effort is that of the players on the team, how these efforts are combined as well as how the individual players and teams are nurtured, developed, and coached by managerial and coaching staff also clearly matter. This indicates that the ultimate "effort" of a sports team can also be best described by a production function that includes many inputs. For wars, the efforts of the adversaries can be thought of as military capacities in the battlefield that are themselves outputs of different types of labor and arming (themselves produced with other inputs).

Contest functions are probabilistic choice functions that, to our knowledge, were first proposed by Luce (1959) in order to study individual choice. Later, and somewhat independently, econometricians developed such functions for the estimation of discrete choice variable (e.g., McFadden, 1974). Friedman (1958) is an early application of the popular "ratio" functional form to an advertising game.

In this paper we first review the different functional forms that have been employed in applications of contests and show how some of them can be derived using four different methods. First, *stochastic* derivations of contest functions start from the supposition that effort is a noisy contributor to some outputs and comparison of the different outputs of players determines the outcome of the contest. The probit and logit forms are the two most well-known and used forms that can be derived stochastically. Second, *axiomatic* derivations link combinations of properties (or, axioms) of contests to functional forms. The logit form can also be derived axiomatically as a special case of the more general additive form. Third, *optimal-design* derivations suppose that a designer with certain objectives about effort or other variables designs the contest, with the functional form being a result of such a design. Finally, *positive-microfoundations* derive contest functions by characterizing environments in which they naturally emerge as win probabilities of the contestants instead of being consciously chosen by a contest designer. We review incomplete information, search-based and Bayesian representations. By no means do all derivations relate to the different environments to which contests have been applied and we will be indicating the areas of applications that each derivation is better suited for. We also review some econometric issues in the econometric estimation of contest functions.

In the next section, we review the different classes of functional forms that have appeared in the literature and discuss some of their properties. In Section 3 we explore the four different types of derivations of contest functions, in Section 4 we examine some issues in estimation, and we conclude in Section 5.

2. Probit, logit and other functional forms

Our purpose in this section is to introduce and discuss the properties of different functional forms of contest technologies before exploring their theoretical foundations in the next section.

Consider two adversaries or contestants, labeled 1 and 2. Denote their choice of efforts as e_1 and e_2. We suppose that efforts are themselves outputs of production functions of different inputs as discussed in the introduction. These production functions can be the same for the two adversaries or they can be different. Associated with them are cost functions $c^1(e_1)$ and $c^2(e_2)$. Since we are solely concerned with how pairs of efforts translate into probabilities of wins and losses and not how efforts might be chosen, we will keep these cost and production functions in the background. For any given combination of efforts, each rival has a probability of winning and a probability of losing. Denote the probability of party $i=1$ winning as $p_1(e_1, e_2)$ and the probability of party $i=2$ winning as $p_2(e_1, e_2)$.

For the p_i's to be probabilities, they need to take values between zero and one, and add up to one: $p_2(e_1, e_2)=1\text{-}p_1(e_1, e_2)\geq 0$. Moreover, we can expect an increase in one party's effort to increase its winning probability and reduce the winning probability of its opponent; that is, we should have $p_1(e_1, e_2)$ strictly increasing in e_1 (when $p_1(e_1, e_2)<1$) and strictly decreasing in e_2 (when $p_1(e_1, e_2)>0$).

A class of functions that has been widely examined takes the following additive form:

$$p_1(e_1, e_2) = \begin{cases} \dfrac{f(e_1)}{f(e_1)+f(e_2)} & \text{if } \sum_{i=1}^{2} f(e_i) > 0; \\ \dfrac{1}{2} & \text{otherwise,} \end{cases} \tag{1}$$

where $f(\cdot)$ is a non-negative, strictly increasing function. This class has been employed in a number of fields, including in the economics of advertising (Schmalensee, 1972, 1978), sports economics (Szymanski, 2003), rent-seeking (Nitzan, 1994; Tullock, 1980), as well as contests in general (Konrad, 2009).

One unique and appealing feature of the class of contest functions in Eq. (1) is that it naturally extends to contests involving more than two parties. Thus, if there were n parties to the contest, denoting the effort of rival i by e_i, and the vector of efforts by all other agents $j\neq i$ by e_{-i}, the winning probability of i would be as follows:

$$p_i\ (e_i, e_{-i}) = \begin{cases} \dfrac{f(e_i)}{\sum_{j=1}^{n} f\left(e_j\right)} & \text{if } \sum_{j=1}^{n} f\left(e_j\right) > 0; \\ \dfrac{1}{n} & \text{otherwise.} \end{cases} \tag{2}$$

The most commonly used functional form is the one in which $f(e_i)=e_i^{\mu}$,[3] where $\mu>0$ (and often, for technical reasons of existence of pure-strategy Nash equilibrium, $\mu\leq 1$), so that

$$p_1(e_1, e_2) = \frac{e_1^{\mu}}{e_1^{\mu}+e_2^{\mu}} = \frac{\left(\frac{e_1}{e_2}\right)^{\mu}}{\left(\frac{e_1}{e_2}\right)^{\mu}+1}. \tag{3}$$

This functional form, sometimes referred to as the "power " form or as the "ratio " form, is that which was employed by Tullock (1980) and the ensuing voluminous literature on rent-seeking. This is also the workhorse functional form used in the economics of conflict. As Hirshleifer (1989) has noted, the probability of winning in this case depends on the *ratio* of efforts, $\frac{e_1}{e_2}$, of the two parties.

A suitable modification of Eq. (1) can accommodate asymmetric effects of contestant efforts on the win probabilities as shown by the following functional form, where $f_i(\cdot)$ is a non-negative, strictly increasing function:

$$p_i(e_1, e_2) = \frac{f_i(e_i)}{f_1(e_1)+f_2(e_2)}. \tag{4}$$

Assuming $f_i(e_i)=a_i f(e_i)$, a particularly convenient version of Eq. (4) is given by:

$$p_1(e_1, e_2) = \frac{a_1 f(e_1)}{a_1 f(e_1)+a_2 f(e_2)}, \tag{5}$$

[3] A variation on this form is $f(e_i)=ae_i^{\mu}+b$ where $a, b>0$. Amegashie (2006) examined the properties of this form.