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Endogenous strength in conflicts☆

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ABSTRACT

In this paper we study a two period contest where the strength of players in the second period depends on the result of the contest in the first stage. We show that in contrast to one-shot contests in the same setting, heterogeneous players exert different efforts in the first stage and rent dissipation in the first period may be large. We study the conditions under which the discouragement effect holds. In addition, new issues emerge like the evolution of the strengths and the shares of the prize during the game.

1. Introduction

The theory of contests analyzes situations in which several contenders expend effort to win a prize. The theory developed from the initial papers by Tullock (1967), Krueger (1974) and Becker (1983), see also Hirshleifer (1991), assumed in the main that the effort of different players had an identical impact in the contest. We will refer to this impact as the strength of a player. Static models in which players have different strengths were considered by Hillman and Riley (1989), Gradstein (1995), Corchón (2000) and Cornes and Hartley (2005).

Dynamic contests have been studied in a number of papers focusing on infinite horizon models (Cairns, 1989; Leininger and Chun-Lei, 1994; McBride and Skaperdas, 2007; Wirl, 1994), two period models of war and settlement (Garfinkel and Skaperdas, 2000; Skaperdas and Syropoulos, 1996) and models in which players have to win a number of contests in order to win a grand contest (Konrad and Kovenock, 2009; see also the surveys of Konrad, 2009, Chpt. 8, and Konrad, 2010). All these papers assume that the strength of players does not vary during the contest.

In this paper we present a two period, two players contest in which the strength of players is endogenous. The contest in each period is modeled by an asymmetric Tullock contest success function (CSF) where effective effort in the contest is determined by the strength of the player and her effort. At the end of each period, players receive their share in the contested resource. This departs from the usual interpretation of a CSF in which the outcome of the contest is probabilistic.

We assume that the strength of a player in the second period depends on the share obtained in the first period. This assumption captures situations such as wars in which the strength of a country depends on the fraction of the territory owned by this country. Another example might be the cold war between the USSR and the US in which the relative strength of each side could be measured by the territories (or the population) under its control. Also a firm with greater market share today could build its "brand" for the future; and a team that wins today can receive more money that will, in turn, make it more competitive in the future. Finally, in a political campaign, the first period contest is a poll which determines the strength of the two candidates in the election.

We prove the existence of a Subgame Perfect Nash Equilibrium which, under some additional assumptions, is unique. In equilibrium, the player with the largest relative strength exerts the largest effort. The latter does not hold in one shot games with two players and Tullock CSF where players with different strength exercise the same effort. Relative strengths count here because the second period

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creates different incentives for players with different relative strengths. We show that the ratio of the effort of player 1 with respect to player 2 in period 1 is increasing in the strength of player 1. Thus, when the effort in the first period is also an investment for the second period, the stronger player exerts more effort in both absolute and relative terms than the weaker player.

The previous properties prompt us to compare the effort made in the first period of our game with the effort made if the game were one shot. This issue has been studied in several papers and discussed in Konrad (2010). In many cases, multi-stage contests involve a "discouragement effect" in which weak players exert less effort in early stages than they would if the contest were one shot. We find that the discouragement effect also holds in our framework when the weak player is sufficiently weak. But it does not always hold. Even if a player is three times stronger than the other the latter exerts more effort than in a one shot game. This is because in our framework players receive a prize in each period and not only at the end of the grand contest.

The Matthew effect is the phenomenon where "the rich get richer and the poor get poorer". To study this effect in our model, we distinguish between the trajectory of strength and the dynamics of the share of the prize. When the link between periods is strong (no discount and the strength in the second period equals the share in the prize in the first period) an initially strong player will be even stronger in the second period. We call this the "avalanche effect" because the initial advantage of a player is amplified later on. However when the link between periods is not strong the avalanche effect only occurs when initial strengths are similar. When initial strengths are unequal the relative strength of the strong player decreases in the second period. We call this the "level-off" effect. It is caused by an increase in the relative effort of the weak player. When the link between periods is weak the avalanche effect disappears, so in the second period relative strengths are leveled off with respect to what they were in the first period.

The trajectory of the share of the prize, does not follow the behavior of strengths: the player having initially more than half of the prize ends having a smaller share in the second period than in the first one. This is because the trajectory of the prize is determined by two forces. First, in the second period both players exert the same effort and therefore their shares coincide with their strength in this period. And two, the transition function is a contraction which means that it translates the impact of shares on strength in a moderate way.

Finally we study rent dissipation. We show that only when players have identical initial strengths and the link between periods is the strongest, rents are completely dissipated. When players are very similar and the link between periods is strong, there is more rent dissipation in the two period game than in the one shot game. But rent dissipation is not monotonic with the link between periods. Weak links can be associated with more rent dissipation than strong links due to the discouragement effect.

There are papers which also endogenize the strength of the players, see Nti (2004) and Franke et al. (2009) for a model where the strength is chosen by a planner. In other papers the CSF is not determined by a planner. Fearon (1996) (see also Leventoglu and Slantchev, 2007) presented a model in which the bargaining power is endogenous and determined by the size of the territory and the threat of a war in which one of the countries would disappear. In our model there is no final battle but a protracted conflict like in the multi-battle models.

The closest paper to ours is by Klumpp and Polborn (2006). In their model, candidates to office have to win a certain number of elections in order to win the grand contest. They show that the outcome of the first election creates an asymmetry in later rounds which might be decisive for the grand contest. They provide an explanation based on rational players for the "momentum effect" which is the tendency of early winners in preliminary contests to win the grand contest. The main difference with our paper is that the prize is obtained at the end of the grand conflict and that the strengths of players are exogenous. In their case the expected value of the prize at each moment is the variable which changes as the game is unfolding.

The rest of the paper goes as follows. Section 2 presents the model. Section 3 gathers our results on the existence and the uniqueness of equilibrium. The properties of equilibrium are shown in Section 4. Section 5 concludes.

2. The model

2.1. Players and payoffs

Two players, $i \in \{1,2\}$, fight for a divisible prize in two periods, $t \in \{1,2\}$. Each player ends each period with a fraction p_i^t of the prize. The value of the prize for each player in each period is V. The interpretation is that the resource under conflict produces a certain surplus each period that can be expropriated by the owner (harvest, money, slave population, human capital, etc.) and that this surplus does not depend on the intensity of conflict.

Player i exerts an effort e_i^t in period t. We assume that the marginal cost of effort is constant and equal to 1. Payoffs in period t are denoted by π_i^t and equal $p_i^t V - e_i^t$, $i \in \{1,2\}$. Payoffs for the whole game are $\sum_{t=1}^{2} \delta^{t-1} \pi_i^t \equiv \Pi_i$ where $\delta \in [0,1]$ is the discount rate of the players.

Players have relative strengths which determine the impact of their effort. We denote by $\alpha^t \in [0,1]$ the relative strength of player 1 at t, and by $1-\alpha^t$ the relative strength of player 2 at t. The contest success function (CSF) maps efforts and strengths in a period into the fraction of the prize owned by the players in this period. This departs from the usual interpretation of the CSF in which the outcome of the conflict is a probability of winning it. Let p (resp. $1-p$) be the fraction obtained by player 1 (resp. 2). We assume the CSF takes the asymmetric general Tullock form:

$$p^t = \frac{\alpha^t \left(e_1^t\right)^\gamma}{\alpha^t \left(e_1^t\right)^\gamma + \left(1-\alpha^t\right)\left(e_2^t\right)^\gamma} \text{ if } e_1^t > 0; \quad p^t = \alpha^t \text{ otherwise.} \tag{2.1}$$

$$1-p^t = \frac{\left(1-\alpha^t\right)\left(e_2^t\right)^\gamma}{\alpha^t \left(e_1^t\right)^\gamma + \left(1-\alpha^t\right)\left(e_2^t\right)^\gamma} \text{ if } e_1^t + e_2^t > 0; 1-p^t = 1-\alpha^t \text{ otherwise.} \tag{2.2}$$

The parameter γ measures the sensitivity of the probability of winning to the efforts. When $\gamma = 0$, the outcome of the contest is independent of efforts. When $\gamma = 1$, the CSF is *proportional*. It seems reasonable to require that the CSF is homogeneous of degree zero, so winning probabilities do not depend on how resources are measured (euros or dollars, thousands or millions of soldiers, etc.). Clark and Riis (1998), following Skaperdas (1996), have shown that under certain assumptions the only functional form that is homogeneous of degree zero is precisely the one above.

Efforts and relative strength enter multiplicatively in the CSF. Think of the relative strength as capital (social or physical) or territory and of $\alpha^t (e_i^t)^\gamma$ as the (Cobb–Douglas) production function of the influence of player i in the contest. Thus influence in the contest is produced by capital and labor. This interpretation of the influence of a player in the contest as a production function that depends of multiple inputs was already pointed out by Nti (2004), Kolmar and Wagener (2005), Cornes and Hartley (2005) and Ray and Sarin (2009).

Finally, note that the only source of asymmetry among players in payoffs and strategies comes from relative strength in period one which is exogenously given.