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MWTmat—application of multiscale wavelet tomography on potential fields

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ABSTRACT

Wavelet analysis is a well-known technique in the sciences to extract essential information from measured signals. Based on the theory developed by previous studies on the Poisson kernel family, this study presents an open source code, which allows for the determination of the depth of the source responsible for the measured potential field. MWTmat, based on the Matlab platform, does not require the wavelet tool box, is easy to use, and allows the user to select the analyzing wavelets and parameters. The program offers a panel of 10 different wavelets based on the Poisson kernel family and the choice between a fully manual and a semiautomatic mode for selection of lines of extrema. The general equations for both horizontal and vertical derivative wavelets are presented in this study, allowing the user to add new wavelets. Continuous wavelet analyses can be used to efficiently analyze electrical, magnetic, and gravity signals; examples are presented here. The MWTmat code and the multiscale wavelet tomography approach are an efficient method for investigating spatial and temporal changes of sources generating potential field signals.

1. Introduction

Since the 1980s, continuous wavelet transforms have become an important tool for signal analyses. In the late 1990s, the ground-breaking work of Moreau et al. (1997, 1999) enhanced our understanding of the sources responsible for potential field signals (i.e., gravity, magnetism, and electricity) by creating the Poisson kernel family, which enables depth calculation of the source of the measured signal. While analyses based on traditional wavelets (i.e., Morlet, Mexican hat) became more widespread in the sciences (Grossmann and Morlet, 1984; Goupillaud et al., 1984; Tchamitchian, 1989, and references therein), the Poisson kernel family has had only limited use in geosciences for potential field data. However, numerous studies have shown the importance of the Poisson kernel family in both real and complex continuous wavelet transforms (e.g., Saracco, 1994; Moreau et al., 1997, 1999; Sailhac et al., 2000; Sailhac and Marquis, 2001; Fedi and Quarta, 1998; Martelet et al., 2001; Saracco et al., 2004, 2007; Boukerbout and Gibert, 2006; Cooper, 2006; Fedi, 2007; Mauri et al., 2010). In this study, the continuous wavelet transform was chosen over other techniques (e.g., wavenumber decomposition) because of its capacity to simultaneously perform multiscale analysis, depth determination, and homogeneous distribution of the source without a priori source information.

This study presents an open source user friendly Matlab code, MWTmat, for real and complex wavelet analyses on potential fields, which allows the user to locate the sources of electrical (self-potential), gravity, or magnetic signals. The code uses a panel of 10 different wavelets based on the Poisson kernel family that enables one to study the depth and structure coefficient of the source of analyzed signal (Fig. 1). The depth calculation method is based on a statistical approach, which allows one to both limit artifact depth and reinforce the localization and the homogeneous distribution of the source by cross-correlation of the calculations using different wavelets. A brief overview of the mathematical background of Poisson kernel family wavelets is presented along with examples from both synthetic and field studies of self-potential, magnetic, and gravity signals. Finally, the multiscale wavelet tomography (MWT) approach is discussed with its application to potential field source localization. In this study, we define complex analyses to be the result of the depth calculation on both the real and the imaginary values that result from the wavelet analyses.

2. Continuous wavelet transform

The continuous wavelet transform (CWT), $L_{(b,a)s}$, is the conversion of any signal into a matrix made of a sum of scalar products in Fourier space, which can be seen as how well the signal matches the analyzing wavelet (Fig. 1). As both analyzed signal and analyzing wavelet have their own signature (e.g., shape, structure, and amplitude), the analyses of the first by the second give a unique signature, which allows characterization of the

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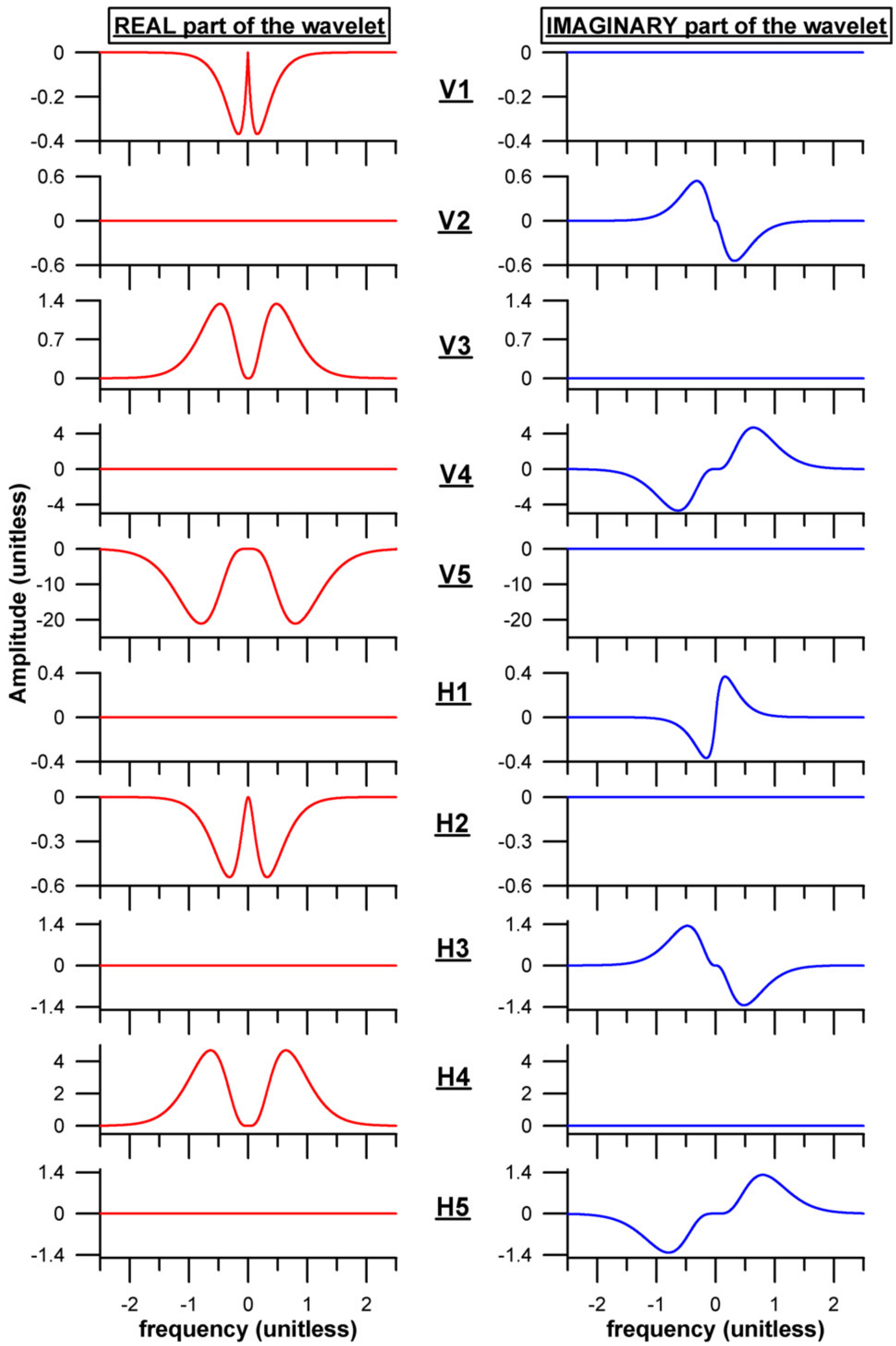

Fig. 1. Poisson kernel wavelet family in Fourier space with their real and imaginary parts. V1 to V5 are the vertical derivatives of order from 1 to 5. H1 to H5 are the horizontal derivatives of order from 1 to 5. Each wavelet is calculated over 1024 points on a frequency from 0 to 2.5 at a dilation $a=1$. The negative part of the frequency axis is the symmetrical construction to give the wavelet its full shape.

structure of the analyzed signal (e.g., frequency content and structure; Fig. 2). The mathematical expression of the wavelet transform, $L_{(b,a)}$, for a signal, s, by a wavelet, g, can be described as follows (Grossmann and Morlet, 1984; Moreau et al., 1997):

$$L_{(b,a)s} = a^{-\gamma} \int_X g_n([x\text{-}b]/a)s(x)dx^{\gamma} \quad \text{with} \quad q = n+\gamma+a, \tag{1}$$

where the dimension order of the space, $\gamma \in N$, b is the translation parameter, and a the dilation parameter; this allows the analyzing wavelet to act as a band filter. The order of the derivative, $n \in N$, signal s has a homogeneous distribution order, $\alpha \in N$, and size of the signal, $x \in N$. X represents the number of elements making the analyzed signal.

These studies apply the CWT within the frequency domain, rather than within the spatial domain, for increased efficiency. Within the frequency domain, the general equation of the horizontal derivative of order n of the Poisson kernel family, $H_n(u)$ (Moreau et al., 1997, 1999; Saracco et al., 2004), is

$$H_n(u) = (2\pi u)^n \exp(-2\pi|u|), \tag{2}$$

with u the wavenumber of the spatial variable, x, in the frequency domain and n being the order of the derivative, such as $n \in N$.