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Gambling by auctions

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ABSTRACT

We provide theoretical and empirical analysis of a selling mechanism used by an Internet web-site that combines important features of auctions and gambling. This is the first analysis of such a selling mechanism, which provides insights into how the two kinds of behavior might be related in real life. The winner of the object is the bidder with the highest bid not submitted by any other bidder. In the equilibrium of our game theoretical model, each bid made with positive probability yields the same probability of winning. Bidders are more likely to submit higher bids, and the bid distribution does not depend on the value of the object or the highest bid allowed if one controls for the number of bidders. Most of these key theoretical predictions are confirmed by the data.

1. Introduction

Two of the most common non fixed-price mechanisms that allocate objects to individuals are auctions and lotteries. In auctions the probability that player *i* wins depends on the other bids, as well as the payment size. In a lottery all agents have the same probability of winning the object, and the actions of the other players might affect the winning prize (for example, when there is more than one winner the winning prize will be divided equally) but do not affect the probability of winning. In this paper we conduct – both theoretical and empirical – analysis of a selling mechanism that combines elements of an auction and a lottery. The mechanism studied is used by the internet portal www.auctions4acause.com, which provided data on its auctions. Before each auction, the auctioneer determines three parameters of the auction: the highest bid allowed (which is less than 10% of the retail value of the object), the maximum number of bids allowed before the auction closes, and the entry fee each bidder needs to pay when submitting his bid. All of these values are made public before the bidding starts. After the bidders pay the participation fee, they submit sealed bids, less than or equal to the highest bid allowed. The winning bid is the highest unique bid (in the sense that no one else bid exactly the same amount) among all bids received. The winner then pays his bid price and obtains the object.

We call the selling mechanism adopted by the portal a Gambling Auction, because it has features that make it a combination of an auction and a lottery. First, the bid and the probability of winning are not monotonically related, because a lower bid might well win the auction if many bidders are placing high bids. Consequently there is no obvious bid that maximizes the probability of winning and, as we show, in equilibrium all bids provide approximately the same probability of winning. Second, this mechanism is not a pure lottery either because the winning probability is determined by the action of the bidders and not by an exogenous randomizing device: the winner is the one who submits the highest unique bid. Note that, under the symmetric Nash equilibrium of the game, the (approximately) equal winning probabilities that this auction creates and the expected payments can be approximated by using a lottery. Thus, the two types of mechanisms are approximately outcome equivalent if the bidders are risk neutral and follow the symmetric equilibrium.[1]

The theoretical analysis finds that in a symmetric equilibrium each bidder chooses his bid using a distribution function over a support that has no gap. This equilibrium strategy is increasing; namely the probability of placing a higher bid is not less than that of a lower bid. The

[1] This Gambling Auction is also interesting, because it can be used in countries or U.S. states that forbid gambling, because the rules of the mechanism do not meet the traditional definitions of a lottery. The mechanism might attract people who like participating in gambling activities, since at a relatively low cost they have the opportunity to win a sizable prize. The auctioneer will make more money using this mechanism than by regular auction mechanisms if participants are risk lovers. Empirically, this is the case since these auctions have a negative expected profit for a bidder. This mechanism is similar to rotating saving and credit associations (roscas) in which group of people save for indivisible good. Each period all the people contribute to the rosca and it is given to someone randomly that is able to get the good. In the next period it is given to somebody else and so on (see Besley et al., 1993). In our mechanism the good is also eventually distributed randomly and each individual pays the participation fees, but the expected payoff is negative, since the auctioneer obtains a positive profit and the winner pays extra money (the winning bid) in order to get the good.

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intuition is that otherwise a higher bid would make winning more likely and thus be more profitable than a lower bid, which would make everyone prefer it, destroying the alleged equilibrium bidding pattern.

We test this prediction with a novel data set collected from the portal www.auctions4acause.com, which implements auctions as described above. The data confirms that the probability of a higher bid is not less than a lower bid. We also find that an increase in the number of bidders increases the number of bids for a given slot, while reducing the probability that each bidder places his bid at this given slot. This leads to an increase of the distance between the maximum bid allowed and the actual winning bid.

We also tested the theoretical prediction that each bid has the same probability of winning by constructing a frequency table (Table 4). This table measures the frequency with which the highest bid wins by calculating the number of auctions in which the highest bid won divided by the number of instances in which a highest bid was placed. We repeat this exercise at lower bid levels and ask whether the empirical frequencies are indeed equal as suggested by the theory. Both formal chi-square tests and informal analysis suggest that the theoretical bid distribution is not consistent with the data.

We also relate our findings to the literature on lotteries. Unlike other studies that find that consumers respond to expected returns, we find that consumer demand for this lottery is not sensitive to the expected payoff but is sensitive to the size of the prize.

Recently, several papers analyzed different types of unique bid auctions as well. Unlike our paper, Eichberger and Vinogradov (2007) examine a least unmatched price auction in which the winner is the lowest unique bidder. Similar to our results, using data from Germany and AuctionAir, they found that on average people tend to place low bids, as the theory predicts. The actual frequency of low bids was higher than the theoretical prediction, however. Rapoport et al. (2007) also construct symmetric mixed-strategy equilibrium for both the highest and lowest unique bid and test their model predictions using an experiment. Meanwhile, Östling et al. (2007) solve a similar game in which the winner is the one who picks the lowest unique integer. The difference of this paper from our paper is that players did not have to pay their bids if they won and the number of bidders was not fixed. Using both a field data (from the Swedish lottery) and experimental data, they test the model predictions and demonstrate that bidders learn to approximate the Poisson equilibrium outcome.

The paper is organized as follows. In the next section we characterize the equilibrium strategies of the auction game and provide some comparative static results. Section 3 discusses and extends the baseline model. Section 4 describes the data, while Section 5 performs empirical analysis. A final section offers some concluding remarks.

2. Theoretical considerations

We will first describe the model we consider and then show that in a symmetric equilibrium a higher bid is chosen with higher probability.

There are $k \geq 3$ bidders[2] who all value the object at the retail price, v. After paying an entry fee of c each bidder submits a sealed bid that is less than a maximum value $b << v$. We assume that each bidder places only one bid. There is a minimum bid increment, which we normalize to 1. The winner is the one who placed the highest bid that was not bid by anyone else.[3] The internet portal reports that, in the rare event of no unique bid, the bidders will be notified about the situation and asked to submit a new bid without additional charge. The winner has to pay an amount equal to his bid, while the losers only pay the entry fee. In addition we assume that k, v and b are such that in equilibrium the winning bid is close to b; in other words, we assume that the bid increment is low compared to the value of the object, and thus the winning bid is close to the maximum allowed bid b.[4] Under such conditions we make the simplifying assumption that each bidder is interested in maximizing his probability of winning the object, ignoring the payment consequences of his bid.[5] The entry fee is already sunk at the bidding stage, so it does not affect bidding strategies.

First, note that the above game has an equilibrium, since after imposing a minimum bid requirement of 0, the auction becomes a finite game. We concentrate on symmetric equilibria, since bidders are symmetric and it is our conjecture that asymmetric equilibria do not exist anyway. Using Kakutani's fixed point theorem we may also show that symmetric (mixed strategy) equilibrium exists. To see this, let n be the number of possible bids, i.e. the number of integers less than or equal to the maximum possible bid plus 1. If all the other bidders use the same mixed strategy $p = (p_1,\ldots,p_n)$, then one can define the set of best replies of bidder i as $BR\ (p)$. This correspondence satisfies the condition of Kakutani's theorem and thus a fixed point exists, which is a symmetric mixed strategy equilibrium of our game.

Claim 1. *In any symmetric equilibrium there is no gap in the support of the equilibrium strategy.*

Proof. Suppose there was a gap at b', i.e. bidder i assigns positive probability to bid $b' - 1$, but assigns a zero probability to bid b'. Then bidding b' would strictly dominate bidding the next available bid b' − 1, which yields a contradiction in that b' − 1 is in the support of the equilibrium strategy. □

Note, that the above claim also implies that the high end of the support is the maximum allowed bid, b. Then a symmetric equilibrium is characterized by the number of bids employed, n, and the probabilities of each of those bids, $p_i = \Pr(b - i + 1)$ where $i = 1,\ldots,n$.

Theorem 1. In a symmetric equilibrium the probability of a higher bid is not less than a lower bid: $i > j$ implies that $p_i \leq p_j$. Moreover, $p_i = p_j$ can hold only when there are four bidders. In that case, the unique equilibrium has $p_1 = p_2 = 1/2$.

Proof. See the Appendix A. □

The intuition behind this result is clear. Suppose that the other bidders randomize equally among the bids $B = \{b_1, b_2, \ldots, b_n\}$, where $b_1 > b_2 > \ldots > b_n$. Then it is easy to see that if bidder i places the bid b_1, then he has a higher probability of winning than with any other bid that belongs to B. But this yields a contradiction, because in a symmetric equilibrium bidder i uses a mixed strategy with support on B, and thus he is indifferent between any of the bids belonging to B. The incentive to bid high is eliminated only if a bidder expects that there are more bidders who placed a high bid than who placed a lower one. Thus, in equilibrium each bidder must place a higher bid with higher probability.

Unfortunately, an explicit solution is not available for the general case of k bidders. Therefore, let us consider some examples with a small number of bidders. First, if there are three bidders, then, in the unique equilibrium all the bids down to zero are used. With T possible bids including 0 it holds that for all $1 < i < T$, $p_i = 1/2^{T-i}$ and $p_0 = 1/2^{T-1}$ is the unique symmetric equilibrium of the game. If $k = 4$, it is easy to show

[2] In the auctions at the above website only the maximum number of bidders is specified, but the number of actual bidders is usually close to the allowed maximum number of bidders, so one may assume that the number of bidders is a known constant, k.

[3] If there is no such bid, then we assume that the seller runs the auction again with the same set of bidders immediately so that there is no discounting between runs of the auction. As one can see, the auction will end with a fixed probability q in any given rerun, if it has not ended before. To avoid considering a game with potentially infinitely many repetitions, we just replace future winning probabilities with $1/k$, that is with the equilibrium winning probability. This transforms the game into a static game with a payoff of $1/k$ in case of a complete tie.

[4] On average, the distance between the winning bids and the maximum allowed bid in our data is less than 14 cents, and the maximum distance is less than $1.5.

[5] We discuss the game where payoff consequences are taken seriously in Section 3. We show for numerical examples that the difference this modification causes in equilibrium strategies is very small.