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Dynamic Surface Control for Non-Strict-Feedback Stochastic Nonlinear Interconnected Systems

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***Abstract*—In this paper, a neural-network-based dynamic surface control method is developed for a class of non-strict-feedback stochastic nonlinear interconnected systems. Neural-networks (NNS) combined with adaptive backstepping technique are applied to model the unknown nonlinear functions of the stochastic interconnected system. The dynamic surface control (DSC) method is adopted to ensure the computation burden is greatly reduced. The proposed controllers guarantee the closed-loop interconnected stochastic nonlinear system is globally bounded stable in probability.**

I. Introduction

With the rapid development of science and technology, stochastic nonlinear interconnected systems have made remarkable achievements [1]. The interconnected system is a large-scale system that is made up of correlative and interactive subsystem, which universally used in practical engineering, such as networked control systems, chemical systems, power systems, vehicle control systems, etc. There are many results on large-scale interconnected systems in the literature, for instance, the decentralized stabilization problem of linear, time-invariant, large scale interconnected systems was studied in [2], the problem of integrated fault estimation and fault-tolerant control for interconnected linear systems with uncertain nonlinear interactions subject to unknown bounded sensor faults [3] was addressed.

On the other side, adaptive backstepping control technology plays a very important role in the nonlinear control systems. The design method can effectively solve the problem of stabilization and tracking control for nonlinear systems with lower triangular form [4–7]. Furthermore, from the fact that neural network (NN) has a strong approximation ability [8, 9], it can be used to solve the control problem for a large number of classes of the nonlinear systems (contain completely unknown functions) under the design framework of backsteping.

For the design of nonlinear non-strict-feedback system is still an open research topic, this paper studies the adaptive neural control problem for a class of uncertain non-strict-feedback stochastic nonlinear interconnected systems by used neural network, backsteping, and dynamic surface control (DSC) technique. First, the observer will be introduced to estimate the unavailable states. Then, based on lyapunov function and the backstepping method, the controller combined with the adaptive law is designed such that all the signal in the closed-loop system are semi-globally uniformly ultimately bounded in mean square.

II. Problem formulation and observer design

Consider a class of non-strict-feedback stochastic interconnection systems as follow:

$$\begin{aligned} d\xi_{i,1} &= \left(\xi_{i,2} + f_{i,1}\left(\xi_i\right) + l_{i,1}(\eta)\right) dt + j_{i,1}^T\left(\eta_i\right) dw_i, \\ d\xi_{i,n_i-1} &= \left(\xi_{i,n_i} + f_{i,n_i-1}\left(\xi_i\right) + l_{i,n_i-1}(\eta)\right) dt \\ &\quad + j_{i,n_i-1}^T\left(\eta_i\right) d\omega_i, \\ d\xi_{i,n_i} &= \left(u_i + f_{i,n_i}\left(\xi_i\right) + l_{i,n_i}(\eta)\right) dt + j_{i,n_i}^T\left(\eta_i\right) d\omega_i, \\ \eta_i &= \xi_{i,1}, \end{aligned} \tag{1}$$

where $\xi_i = [\xi_{i,1}, \xi_{i,2}, \cdots, \xi_{i,n_i}]^T \in R^{n_i}$, $u_i \in R$ and $\eta_i \in R$ are the state variable, the system input and the output of the i-th subsystem, respectively; $f_{i,y}(\cdot) : R^n \to R$ and $j_{i,y}(\cdot) : R^i \to R^r$ are unknown smooth nonlinear functions with $f_{i,y}(0) = 0$ and $j_{i,y}^T(0) = 0$; $l_{i,y}(\eta)$ are the interconnected terms, for any $i = 1, 2, \cdots, N, y = 1, 2, \cdots, n_i - 1$; ω_i denotes an independent r-dimensional standard Wiener process.

Assumption 1.1: For $1 \leq i \leq N$, $1 \leq y \leq n_i$, there is a positive unknown constant $m_{i,y}$, such that $\left|l_{i,y}(\eta)\right| \leq m_{i,y} \sum\limits_{s=1}^{N} \phi_{iys}(|\eta_s|)$.

Assumption 1.2: For the function $f_{i,1}(\cdot)$, there exists an unknown constant R_0, such that the inequality $\left|f_{i,1}\left(\bar{\xi}_i\right) - f_{i,1}\left(\hat{\bar{\xi}}_i\right)\right| \leqslant R_0 \left\|\bar{\xi}_i - \hat{\bar{\xi}}_i\right\|$ holds for $\forall \bar{\xi}_i, \hat{\bar{\xi}}_i \in R^n$.

Assumption 1.3: There exists a class - κ_∞ function $S(\cdot) \in C^2(R^+)$, such that $\left|f_{i,1}(\xi_i)\right| \leqslant S(\|\xi_i\|)$.

Assumption 1.4: For $\forall X_i \in \Omega_{X_i}$, there exists an ideal constant weigh vector ϑ^* such that $\|\vartheta^*\|_\infty \leqslant \iota_{\max}$ and $|\kappa| \leqslant \kappa_{\max}$ with bounds $\iota_{\max} > 0$ and $\kappa_{\max} > 0$. Form the relevant literature [10], we can obtain

$$\begin{aligned} \vartheta^{*T} S\left(X_i\right) + \kappa &\leqslant \left|\vartheta^{*T} S\left(X_i\right)\right| + |\kappa| \\ &\leqslant \sum_{m=1}^{r} \left|s_m\left(X_i\right)\right| \iota_{\max} + \kappa_{\max} \leqslant e\varphi\left(X_i\right), \end{aligned} \tag{2}$$

where $\varphi(X_i) = \sqrt{(r+1)\left(\sum_{m=1}^{r} s_m^2(\mathrm{X}_i)+1\right)}$ and $e = \max\{\kappa_{\max}, \iota_{\max}\}$. In order to estimate the state of the system (1), the following observer is proposed:

$$\begin{aligned}
&\dot{\hat{\xi}}_{i,y} = \hat{\xi}_{i,y+1} - \gamma_{i,y}\hat{\xi}_{i,1}, \quad y = 1, ..., n_i - 1, \\
&\dot{\hat{\xi}}_{i,n_i} = u_i - \gamma_{i,n_i}\hat{\xi}_{i,1},
\end{aligned} \tag{3}$$

where $\gamma_{i,y}\xi_i$ are constant design parameters and the initial condition is given as $\hat{\xi}_{i,y}(0) = \hat{\xi}_{iy0}$. Define $\hat{\xi} = \left[\hat{\xi}_1, ..., \hat{\xi}_n\right]$, and let $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$ be the observer error, and define $\hat{\xi}_{i,n_{i+1}} = u_i$ and $\tilde{\xi}_{i,n_{i+1}} = 0$. From (1) and (3), we rewrite the observer error dynamics of $\tilde{\xi}_i$ in a compact form as

$$d\tilde{\xi}_i = \left(B_i\tilde{\xi}_i + F_i(\xi_i) + H_i\xi_1 + L_i(\eta)\right)dt + J_i d\omega_i, \tag{4}$$

where $F_i(\xi_i) = [f_{i,1}, f_{i,2}, ..., f_{i,n_i}]^T$, $L_i = [l_{i,1}, l_{i,2}, ..., l_{i,n_i}]^T$, $J_i = [j_{i,1}, j_{i,2}, ..., j_{i,n_i}]^T$, $H_i = [h_{i,1}, h_{i,2}, ..., h_{i,n_i}]^T$ and $B_i = \begin{bmatrix} -h_{i,1} & & \\ \vdots & I_{i,n_{i-1}} & \\ -h_{i,n_i}0 & \cdots & 0 \end{bmatrix}$ and $h_{i,1}, \ldots, h_{i,n_i}$ is to be designed such that B_i is asymptotically stable. That is to say, there exist symmetric positive definite matrices C_i and Q satisfying $B_i^T C_i + C_i B_i = -Q$.

III. Adaptive DSC design and stability analysis

A. Observer-based adaptive DSC design

The whole system formed by the system (1) and the observer (3) can be written as follows

$$\begin{aligned}
&d\tilde{\xi}_i = \left(B_i\tilde{\xi}_i + F_i(\xi_i) + H_i\xi_{i,1} + L_i(\eta)\right)dt + J_i d\omega_i, \\
&d\eta_i = \left(\hat{\xi}_{i,2} + \tilde{\xi}_{i,2} + f_{i,1}(\xi_i) + l_{i,1}(\eta)\right)dt + j_{i,1}^T(\eta_i)\,d\omega_i, \\
&\dot{\hat{\xi}}_{i,2} = \hat{\xi}_{i,y+1} - h_{i,y}\hat{\xi}_{i,1}, \quad y = 2, ..., n_i - 1, \\
&\dot{\hat{\xi}}_{i,n_i} = u_i - h_{i,n_i}\hat{\xi}_{i,1}.
\end{aligned} \tag{5}$$

Furthermore, we defined an unknown constant as $v_i = \max\{e_{i,y}, y = 0, 1, 2, ..., n_i\}$. For starting the control design procedure, we let $e_{i,y}$ being specified later, $\hat{v}_i$ express the estimate of v_i, and $\tilde{v}_i$ express the estimate error between $\hat{v}_i$ and v_i, namely $\tilde{v}_i = v_i$ - $\hat{v}_i$.

The coordinate transformations used in this paper can be written as follows:

$$z_{i,1} = \eta_i, \quad z_{i,y} = \hat{\xi}_{i,y} - \alpha_{iy-1f}, \tag{6}$$

where α_{iy-1f} is the output of a first-order filter with the virtual control function $\alpha_{i,y-1}$ will be presented later. The boundary layer error $\sigma_{i,y}$ is defined as $\sigma_{i,y} = \alpha_{iy-1f} - \alpha_{iy-1}$.

Theorem 3.1: Consider the interconnected non-linear system (1) with observer (3), the feasible adaptive neural DSC method is proposed as

$$\alpha_{i,y} = -\varpi_{i,y}z_{i,y} - \hat{v}_i\varphi_{i,y}(\mathrm{X}_{i,y})\tan h\frac{z_{i,y}^3\varphi_{i,y}(\mathrm{X}_{i,y})}{q_{i,y}},$$
$$1 \leqslant y \leqslant n_{i-1},$$

$$u_i = -\varpi_{i,n_i}z_{i,n_i} - \hat{v}_i\varphi_{i,n_i}(\mathrm{X}_{i,n_i})\tan h\frac{z_{i,n_i}^3\varphi_{i,n_i}(\mathrm{X}_{i,n_i})}{q_{i,n_i}},$$

$$\dot{\hat{v}}_i = \lambda\left(\sum_{y=1}^{n_i} z_{i,y}^3\varphi_{i,y}(\mathrm{X}_{i,y})\tan h\frac{z_{i,y}^3\varphi_{i,y}(\mathrm{X}_{i,y})}{q_{i,y}} - \delta\hat{v}_i\right),$$

where $\varphi_{i,y}(\mathrm{X}_{i,y}) = \sqrt{(r_i+1)\left(\sum_{m=1}^{r_i} s_{im}^2(\mathrm{X}_{i,y})+1\right)}$, $\mathrm{X}_{i,1} = [\eta_i, \hat{v}_i]^T$, $\mathrm{X}_{i,y} = \left[\eta_i, \bar{\hat{\xi}}_{i,y}, \bar{\sigma}_{i,y}, \hat{v}_i\right]^T$ with $\bar{\hat{\xi}}_{i,y} = \left[\hat{\xi}_{i,1}, \ldots, \hat{\xi}_{i,y}\right]^T$ and $\bar{\sigma}_{i,y} = [\sigma_{i,2}, \ldots, \sigma_{i,n_i}]^T$ for $y = 2, \ldots, n_i$, in addition, $\varpi_{i,y}, q_{i,y}, \lambda$, and δ are positive design parameters for $y = 1, \ldots, n_i$.

Lemma 3.1: Let $\hat{\Xi}_i = \left[\eta_i, \hat{\xi}_{i,2}, \ldots, \hat{\xi}_{i,n_i}\right]^T$, $\mathrm{X_i} = [\eta_i, z_{i,2}, \ldots, z_{i,n_i}, \sigma_{i,2}, \ldots, \sigma_{i,n_i}, \hat{v}_i]^T$, $z = [z_{i,1}, z_{i,2}, \ldots, z_{i,n_i}]^T$ and $\bar{\alpha} = [0, \alpha_{i1f}, \ldots, \alpha_{in_i-1f}]^T$, then there exist a positive constant c, satisfying $\left\|\hat{\Xi}_i\right\| \leqslant c\,\|\mathrm{X_i}\|$.

Proof: The proof of the lemma can be found in the literature [10].

Step i, 0: Define Lyapunov function as $V_{i,0} = \frac{\tau}{2}\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)^2$, where τ being a positive parameter. Then, we can obtain

$$\begin{aligned}
\mathcal{L}V_{i,0} =& \tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T\left(B_i^T C_i + C_i B_i\right)\tilde{\xi}_i \\
&+ 2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i F_i(\xi_i) \\
&+ 2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i(H_i\xi_{i,1}) \\
&+ 2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T l(\eta) \\
&+ 2\tau Tr\left\{J_i^T\left(2C_i\tilde{\xi}_i\tilde{\xi}_i^T C_i + \tilde{\xi}_i^T C_i\tilde{\xi}_i C_i\right)J_i\right\}.
\end{aligned} \tag{7}$$

Then, by using the Young′s inequality, we have

$$\begin{aligned}
&2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i F_i(\xi_i) \leqslant 2\tau\left\|\tilde{\xi}_i\right\|^3\|C_i\|^2\|F_i(\xi_i)\| \\
&\leqslant \frac{3}{2}\tau d_0^{\frac{4}{3}}\|C_i\|^{\frac{8}{3}}\left\|\tilde{\xi}_i\right\|^4 + \frac{1}{2d_0^4}\tau\|F_i(\xi_i)\|^4,
\end{aligned} \tag{8}$$

$$\begin{aligned}
&2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i H_i\xi_{i,1} = 2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i H_i\eta_i \\
&\leqslant \frac{3}{2}\tau d_1^{\frac{4}{3}}\|C_i\|^{\frac{8}{3}}\left\|\tilde{\xi}_i\right\|^4 + \frac{1}{2d_1^4}\tau\|H_i\|^4\eta_i^4.
\end{aligned} \tag{9}$$

Based on Assumption 1.1, we can further obtain

$$\begin{aligned}
&2\tau\left(\tilde{\xi}_i^T C_i\tilde{\xi}_i\right)\tilde{\xi}_i^T C_i l_i(\eta_i) \\
&\leqslant \frac{\tau}{2}\sum_{y=1}^{n_i}\sum_{s=1}^{N}\phi_{iys}^4(|\eta_s|) + \frac{3\tau n_i N}{2}m_{i,y}^{\frac{4}{3}}\|C_i\|^{\frac{8}{3}}\left\|\tilde{\xi}_i\right\|^4,
\end{aligned} \tag{10}$$

$$\frac{\tau}{2}\sum_{y=1}^{n_i}\sum_{s=1}^{N}\phi_{iys}^4(|\eta_s|) \leqslant \sum_{y=1}^{n_i}\sum_{s=1}^{N}\mu_{iys}\eta_s^4 + \sum_{y=1}^{n_i}\sum_{s=1}^{N}4\tau\phi_{iys}^4(0)$$

$$\begin{aligned}
&2\tau Tr\left\{J_i^T\left(2C_i\tilde{\xi}_i\tilde{\xi}_i^T C_i + \tilde{\xi}_i^T C_i\tilde{\xi}_i C_i\right)J_i\right\} \\
&\leqslant 3\tau n\sqrt{n}d_2\|C_i\|^4\left\|\tilde{\xi}_i\right\|^4 + \frac{3\tau n^2\sqrt{n}}{d_2}\sum_{l=1}^{n}\left\|j_{i,l}\left(\bar{\xi}_i\right)\right\|^4,
\end{aligned} \tag{11}$$