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SIPPI: A Matlab toolbox for sampling the solution to inverse problems with complex prior information Part 2—Application to crosshole GPR tomography

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ABSTRACT

We present an application of the SIPPI Matlab toolbox, to obtain a sample from the a posteriori probability density function for the classical tomographic inversion problem. We consider a number of different forward models, linear and non-linear, such as ray based forward models that rely on the high frequency approximation of the wave-equation and 'fat' ray based forward models relying on finite frequency theory. In order to sample the a posteriori probability density function we make use of both least squares based inversion, for linear Gaussian inverse problems, and the extended Metropolis sampler, for non-linear non-Gaussian inverse problems. To illustrate the applicability of the SIPPI toolbox to a tomographic field data set we use a cross-borehole traveltime data set from Arrenæs, Denmark. Both the computer code and the data are released in the public domain using open source and open data licenses. The code has been developed to facilitate inversion of 2D and 3D travel time tomographic data using a wide range of possible a priori models and choices of forward models.

1. Introduction

Tomographic inversion is used in many research fields such as geophysics and medical imaging. With this technique, images of an unknown 3D object can be obtained based on indirect observations from outside of the object. One such example is travel time inversion that can for example be used to map the internal velocity structure of the earth, based on recordings of the arrival times of certain seismic phases generated as part of e.g. an earthquake. Another example of a tomographic data set, is that obtained by measuring the travel time delay of a seismic or electromagnetic wave traveling between a source and a receiver. Given such a set of observed travel time data the tomographic inverse problem consists of inferring information about the velocity around and in-between the sources and receivers. It is this latter problem that we will address here using the SIPPI toolbox, which is a Matlab toolbox for sampling the solution to inverse problems with complex a priori information (Hansen et al., this issue).

We will specifically address the problem of first arrival travel time inversion using crosshole ground-penetrating radar (GPR) data. Such travel time data are sensitive to the subsurface variations in electromagnetic wave velocity, that is related to the dielectric permittivity, which is strongly influences by water moisture (Topp et al., 1980). Inversion of such travel time data thus has the potential to map subsurface moisture content.

For linear or weakly non-linear inverse problems least squares based methods are widely applied. Deterministic least squares methods is presented by e.g. Menke (1989), while a probabilistic approach is given by e.g. Tarantola and Valette (1982) and Tarantola (2005).

A probabilistic approach to linear travel time tomography, based on sequential simulation, was proposed by Hansen et al. (2006) and Hansen and Mosegaard (2008) who utilized the equivalence of classical least squares inversion (e.g. Tarantola and Valette, 1982) and kriging (e.g. Journel and Huijbregts, 1978). An application of this approach to crosshole georadar data is given in Nielsen et al. (2010). A related method based on kriging through error simulation (Journel and Huijbregts, 1978), equivalent with the probabilistic least squares approach, was proposed and applied to cross hole GPR tomographys by Gloaguen et al. (2005a,b). Recently this approach was applied for inversion of an anisotropic velocity field (Giroux and Gloaguen, 2012). These methods are only strictly valid for linear inverse problems, and rely on an inherent assumption of Gaussian statistics describing both the noise model and the a priori model. Specifically the a

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priori model must be given in form of a Gaussian a priori model defined by a mean and a covariance model. Choosing such a Gaussian prior model may not be trivial. A number of methods have been developed to estimate this model prior to inverting the data (Asli et al., 2000; Hansen et al., 2008a; Irving et al., 2009; Looms et al., 2010).

For examples of least squares based deterministic tomographic inversion of GPR cross hole data see e.g. Irving et al. (2007) and Dafflon et al. (2011). Examples of stochastic inversion is presented for inversion of time lapse cross hole 1D travel time data by Scholer et al. (2012) and 2D time lapse electrical resistivity data by Irving and Singha (2010). Hansen et al. (2008b) demonstrate an application of the extended Metropolis sampler (Mosegaard and Tarantola, 1995) to a non-linear cross hole tomographic problem, where the a priori model is non-Gaussian and defined by any geostatistical method.

Here we will demonstrate the use of the SIPPI Matlab toolbox for solving the crosshole traveltime tomography inverse problem in a probabilistic framework. Initially we will briefly describe the theory describing different linear and non-linear solutions to the forward problem of computing the travel time delay between a propagating wave traveling between a source and a receiver. Then we will demonstrate how these forward models can be utilized with SIPPI. We will then make use of a reference data set obtained at Arrenæs, North Sealand, Denmark, to demonstrate all the inversion methods available in SIPPI, such as classical least squares estimation and simulation, and sampling methods such as the rejection sampler and the extended Metropolis sampler, see Hansen et al. (this issue).

2. Theory, first arrival travel time computation

The travel time delay of a propagating wave between a source and a receiver can be defined in a number of ways. We will consider methods based on the eikonal equation, 1st order sensitivity kernels and the Born approximation.

2.1. The eikonal equation

The eikonal equation describes the arrival time along a closed curve, $u(\mathbf{x})$, traveling with the speed defined by the velocity field, $m(\mathbf{x})$ (Sethian and Popovici, 1999)

$$|\nabla u(\mathbf{x})| m(\mathbf{x}) = 1 \tag{1}$$

Solving Eq. (1) allows locating the travel time, d, between a source and a receiver along the closed curve. To solve the eikonal equation we make use of an efficient implementation of the multistencil fast marching method proposed by Hassouna and Farag (2007), and made available by Dirk-Jan Kroon[1] under an open source license. This forward model is non-linear and, as the eikonal equation corresponds to a high frequency approximation to the wave equation. Therefore, it is often referred to as the high frequency ray approximation.

2.2. Forward models based on 1st order sensitivity kernels

The travel time d between a source and a receiver can be given by

$$d = \int G(\mathbf{x}) \frac{1}{m(\mathbf{x})} \, d\mathbf{x} \tag{2}$$

where $m(\mathbf{x})$ is the velocity field in which the signal travels. $G(\mathbf{x})$ is the sensitivity kernel that describes the sensitivity of each model parameter (within the Fresnell zone) to the travel time. $G(\mathbf{x})$ can be computed under a wide range of assumptions and thus defines the forward problem of computing the travel time delays in different ways.

2.2.1. Ray based forward model

Using the high frequency approximation to the wave equation results in a sensitivity kernel $G(\mathbf{x})$ that can be described by a ray connecting the source and receiver. Hence, this kernel can be obtained by solving the eikonal equation, which provides the fastest possible forward model. We will refer to this type of forward model as ray based.

2.2.2. Fat ray based forward model

Using a finite frequency (band limited) approximation to the wave equation leads to a sensitivity kernel where the sensitivity of the travel time delay also appears in a zone around the fastest ray path. A number of works have defined sensitivity kernels based on geometrical rules assigning sensitivity within the first Fresnel zone. Forward models based on these types of kernels will be referred to as fat ray based forwards (Husen and Kissling, 2001; Jensen et al., 2000).

2.2.3. Born based forward model

The Born approximation to the wave equation (considering only 1st order scattering) is an exact analytical expression for the sensitivity kernel for a point source, which can be derived for both seismic (Dahlen et al., 2000; Spetzler and Snieder, 2004; Marquering et al., 1999; Liu et al., 2009) and electromagnetic wave propagation (Buursink et al., 2008). The Born approximation also leads to a sensitivity kernel with sensitivity outside the ray approximation (i.e. a fat ray). The Born approximation is only strictly valid for a homogeneous velocity field, but have in practice been used also when the velocity field has relatively small velocity contrasts. For large velocity contrast this method becomes unstable and cannot be used. Forward models based on the Born approximation will be referred to as Born based forward models.

3. Cross hole GPR tomography at Arrenæs

As a case study we will demonstrate the capabilities of SIPPI for solving tomographic inverse problems. The implementation is generally applicable for travel time based tomographic problems, but here we will apply the toolbox to a cross hole GPR tomographic problem.

Initially we will present a 3D data set. Then we will demonstrate how the the different types of forward models have been implemented in `sippi_forward_traveltime` for easy utilization as part of SIPPI. Finally we demonstrate the use of SIPPI to solve the GPR cross hole tomography inverse problem using both linear and non-linear forward models, and simple and more complex a priori models.

3.1. Data: 3D GPR crosshole traveltime data from Arrenæs

As a reference data set we consider a 3D tomographic data set recorded as part of a ground penetrating radar (GPR) cross borehole survey at Arrenæs, North Sealand, Denmark. The data set we use here is identical to data presented by Looms et al. (2010), and is here made available in the public domain.

The observed data are first arrival times of electromagnetic waves propagating from a source location in one borehole to a receiver location in another borehole. Thus, the forward problem consists of estimating the travel time delay caused by the subsurface velocity field, given the recording geometry. The inverse

[1] http://www.mathworks.com/matlabcentral/fileexchange/24531-accurate-fast-marching.