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Stability radii of optimal assembly line balances with a fixed workstation set

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ABSTRACT

For an assembly line, it is required to minimize the line's cycle time for processing a partially ordered set of the assembly operations on a linearly ordered set of the workstations. The operation set is partitioned into two subsets, manual and automated. The durations of the manual operations are variable and those of the automated operations are fixed. We conduct a stability analysis for this problem. First, we derive a sufficient and necessary condition for the optimal line balance to have an infinitely large stability radius. Second, we derive formulas and an algorithm for calculating the stability radii for the optimal line balances. Third, we report computational results for the stability analysis of the benchmark instances. Finally, we outline managerial implications of the stability results for choosing most stable line balances, which save their optimality in spite of the variations of the operation durations, and for identifying the right time for the re-balancing of the assembly line.

1. Introduction

The assembly line consists of m workstations, which are linked by a conveyor belt (or another equipment) moving an in-process product from one workstation to the next at a constant pace. The set V of n assembly operations is fixed. Each workstation needs to perform a specific subset of the operations from the set V within the line's cycle-time. All the m workstations start simultaneously to process their own operations. A partial order on the operation set V arises due to technological and economical considerations, which are represented by the precedence digraph $G = (V, A)$ with the set A of arcs. A Simple Assembly Line Balancing Problem is to find an optimal assembly line balance, i.e. an assignment of the operations V to the m workstations such that the cycle-time is minimal. The abbreviation SALBP-2 for denoting this problem has been introduced by Baybars (1986). The problem SALBP-2 is NP-hard (Gutjahr and Nemhauser, 1964; Wee and Manoj, 1982) since the bin-packing problem is NP-hard and is a special case of the problem SALBP-2, where in the bin-packing problem, the digraph $G = (V, A)$ has no arcs, $A = \varnothing$.

Throughout this paper, it is assumed that the set V consists of two specific subsets of the assembly operations. The non-empty subset $\widetilde{V} \subseteq V$ includes all the manual operations and the subset $V\backslash\widetilde{V}$ includes all the automated operations. The initial vector $t = (t_1, t_2, \ldots, t_n)$ of the operation durations is known before solving the problem SALBP-2. However, for the subset $\widetilde{V} \subseteq V$ of the manual operations $j \in \widetilde{V}$, each duration t_j may vary due to different factors such as the operator skill, motivation, learning effect, etc. In contrast to the manual operations, the duration t_i of each automated operation $i \in V\backslash\widetilde{V}$ is fixed. We assume that $\widetilde{V} = \{1, 2, \ldots, \tilde{n}\}$ and $V\backslash\widetilde{V} = \{\tilde{n}+1, \tilde{n}+2, \ldots, n\}$, $1 \le \tilde{n} \le n$. The vectors of the operation durations are denoted as follows: $\tilde{t} = (t_1, t_2, \ldots, t_{\tilde{n}})$, $\bar{t} = (t_{\tilde{n}+1}, t_{\tilde{n}+2}, \ldots, t_n)$, $t = (\tilde{t}, \bar{t}) = (t_1, t_2, \ldots, t_n)$. Let a subset $V_k^{b_r} \neq \varnothing$ of the set V be assigned to the workstation S_k, where $k \in \{1, 2, \ldots, m\}$. The assignment $b_r: V = V_1^{b_r} \bigcup V_2^{b_r} \bigcup \ldots \bigcup V_m^{b_r}$ of the operations V to the ordered workstations $(S_1, S_2, \ldots, S_m)$, $V_k^{b_r} \bigcap V_l^{b_u} = \varnothing$, $1 \le k < l \le m$, is called a line balance, if the following two conditions hold.

Condition 1. The assignment b_r does not violate the partial order given on the set V by the precedence digraph $G = (V, A)$, i.e. each arc $(i, j) \in A$ implies that operation $i \in V$ is assigned to workstation S_k and operation $j \in V$ is assigned to workstation S_l in a way such that $1 \le k \le l \le m$.

Condition 2. The assignment b_r uses all the m workstations, i.e. the subset $V_k^{b_r}$ is not empty for each workstation S_k, $k \in \{1, 2, \ldots, m\}$.

Let $B(G)$ denote the set of all assignments b_r satisfying

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Condition 1. The subset $B(G, m) = \{b_0, b_1, \ldots, b_h\}$ of the set $B(G)$ consists of all line balances. The cycle-time $c(b_r, t)$ for the line balance b_r with the vector $t = (\tilde{t}, \bar{t})$ of the operation durations is determined as $c(b_r, t) = \max_{k=1}^{m} \sum_{i \in V_k^{b_r}} t_i$, where the sum $\sum_{i \in V_k^{b_r}} t_i := t(V_k^{b_r})$ is a workstation time. The line balance b_0 is optimal with the operation durations $t = (\tilde{t}, \bar{t})$ if it achieves a minimal cycle-time c as follows:

Condition 3. $c = c(b_0, t) = \min\{c(b_r, t): b_r \in B(G, m)\}$.

Note that Condition 2 allows us to restrict a set of the line balances since the set $B(G, m)$ contains the optimal line balance without fail. Let $B(G, m, t)$ denote a set of all the optimal line balances, $B(G, m, t) \subseteq B(G, m)$, with the vector $t = (\tilde{t}, \bar{t})$ of the operation durations. If operation i belongs to the set $V \setminus \widetilde{V}$, its duration t_i is fixed. Without loss of generality, we assume that $t_i > 0$ for each automated operation $i \in V \setminus \widetilde{V}$ since the automated operation with the fixed zero duration has no influence on a solution to the problem SALBP-2. The initial duration t_i is a strictly positive real number $t_i > 0$ for each operation $i \in V$. A value of the duration $t_j > 0$ of the manual operation $j \in \widetilde{V} \subseteq V$ can vary during the assembly line lifespan. The varied duration t'_j may be even equal to zero, which means that the manual operation j from the set

$$\widetilde{V}_k^{b_r} := V_k^{b_r} \cap \widetilde{V} \tag{1}$$

is processed by an additional operator in parallel with the processing of other operations assigned to workstation S_k. Due to the additional operator, the processing of the manual operation j does not increase the workstation time, i.e.

$$t'(V_k^{b_r}) = \sum_{i \in V_k^{b_r}} t'_i = \sum_{i \in V_k^{b_r} \setminus \{j\}} t'_i, \tag{2}$$

where t' indicates the modified vector $t' = (\tilde{t}', \bar{t}) = (t'_1, t'_2, \ldots, t'_{\tilde{n}}, t_{\tilde{n}+1}, t_{\tilde{n}+2}, \ldots, t_n) := (t'_1, t'_2, \ldots, t'_{\tilde{n}}, t'_{\tilde{n}+1}, t'_{\tilde{n}+2}, \ldots, t'_n)$, for which the workstation time $\sum_{i \in V_k^{br}} t'_i$ is calculated. The second equality in (2) is valid because of holding equality $t'_j = 0$. We summarize the above in the following remark.

Remark 1. The initial duration t_i is a strictly positive real number for each operation $i \in V$. A value of the duration $t_j > 0$ of the manual operation $j \in \widetilde{V}$ can vary during the assembly line lifespan. The varied duration t'_j may be equal to zero: $t'_j \geq 0$.

The aim of this paper is to investigate the stability of the optimal line balance with respect to variations $\tilde{t}' \neq \tilde{t}$ of the operation durations. The stability radius $\rho_{b_0}(t)$ of the optimal line balance b_0 is interpreted as a maximum of simultaneous and independent variations $\tilde{t}'$ of the durations $\tilde{t}$ of operations $\widetilde{V}$ without violating the optimality of the line balance b_0, i.e. $b_0 \in B(G, m, t) \cap B(G, m, t')$. A formal definition of the stability radius is given in Section 2.1 along with a sufficient and necessary condition for a zero stability radius. In Section 3, it is shown that the stability radius may be infinitely large, $\rho_{b_0}(t) = \infty$. Formulas for calculating the stability radius $\rho_{b_0}(t)$ for the line balance $b_0 \in B(G, m, t)$ are given in Section 4.1. The calculation of the stability radius is illustrated in Sections 2.2, 3.3 and 4.3. In Section 4.2, it is shown on how to restrict a subset of the set $B(G, m) \setminus \{b_0\}$, which must be compared with the line balance $b_0 \in B(G, m, t)$ for calculating the stability radius $\rho_{b_0}(t)$. An algorithm for calculating the stability radius is presented in Section 5. Section 6 reports the computational results for the stability analysis of the benchmark instances from the old dataset and the recent one (Otto et al., 2013) tested in Morrison et al. (2014), Otto and Otto (2014). In Section 7, the managerial implications are spelled out on how to use the stability results in the assembly industry. Concluding remarks and perspectives are discussed in Section 8.

2. Contributions of this work, previous results, and related literature

The assembly lines are widely used in a mass production for assembling components into final products. An effectively balanced assembly line allows a factory to increase its efficiency via reducing a production cost. Since the production conditions may change over time, the need of a re-balancing of the assembly line may arise from time to time in order to serve customer demands in the competitive market environment. The assembly re-balancing is tedious procedures requiring significant costs and amounts of a manpower (Chen et al., 2004; Chica et al., 2013; Gamberini et al., 2006). It is a stability analysis that can help us to identify the right time for the re-balancing. In spite of its practical importance, the literature on the stability analysis of the assembly line balances is scanty (Chica et al., 2013; Gurevsky et al., 2012, 2013; Sotskov and Dolgui, 2001, Sotskov et al., 2005, Sotskov et al., 2006, 2015). Next, we discuss a concept of the stability radius for the problem SALBP-2 (Section 2.1). Section 2.3 contains a brief literature review of other results and approaches for examining the robustness and stability of the assembly line balances. Contributions of this work are discussed in Section 2.4.

2.1. The stability radius of the optimal line balances for the problem SALBP-2

We study the following question. How much can all or some components of the vector $\tilde{t}$ be simultaneously and independently modified that the line balance b_0, which is optimal for the initial vector $t = (\tilde{t}, \bar{t})$, remains optimal for the modified vector $t' = (\tilde{t}', \bar{t})$ of the operation durations? We study the stability radius of the optimal line balance that is defined similarly to the stability radius of the optimal schedule (Bräsel et al., 1996; Sotskov, 1991). If the stability radius of the line balance $b_0 \in B(G, m, t)$ is strictly positive, then the line balance b_0 remains optimal for all variations t'_j of the operation durations $t_j, j \in \widetilde{V}$, within the ball with this radius and center $\tilde{t}$. On the other hand, if the stability radius of the line balance b_0 is equal to zero, then b_0 may no longer be optimal even for infinitely small variations of the operation durations.

In contrast to a stochastic assembly line (Dong et al., 2014; Erel and Sarin, 1998; Gamberini et al., 2006; Kahan et al., 2009), we do not assume the given probability distribution for the random duration t_j of the manual operation $j \in \widetilde{V}$. Note also that operation durations $t_i, i \in V$, are assumed to be *real* numbers, in contrast to the assumption used by Scholl (1999) and many other authors that the operation durations are *integer* numbers. Let $R^{\tilde{n}}$ denote space of all real $\tilde{n}$-vectors $(t_1, t_2, \ldots, t_{\tilde{n}})$ with the following metric: The distance $d(\tilde{t}, \tilde{t}')$ between vector $\tilde{t} = (t_1, t_2, \ldots, t_{\tilde{n}})$ and vector $\tilde{t}' = (t'_1, t'_2, \ldots, t_{\tilde{n}}')$ is defined as $d(\tilde{t}, \tilde{t}') = \max\{|t_i - t'_i|: i \in \widetilde{V}\}$, where $|t_i - t'_i|$ is the absolute value of the difference $t_i - t'_i$. Let $R_+^{\tilde{n}}$ denote space of the non-negative real $\tilde{n}$-vectors, $R_+^{\tilde{n}} \subseteq R^{\tilde{n}}$.

Definition 1. The ball $O_\rho(\tilde{t})$ in space $R^{\tilde{n}}$ with the radius $\rho \in R_+^1$ and the center $\tilde{t} \in R_+^{\tilde{n}}$ is called a stability ball of the line balance $b_0 \in B(G, m, t)$ if for any modified vector $t' = (\tilde{t}', \bar{t})$ of the operation durations with $\tilde{t}' \in O_\rho(\tilde{t}) \cap R_+^{\tilde{n}}$, the line balance b_0 remains optimal. The maximal value $\rho_{b_0}(t)$ of the radius ρ of the stability ball $O_\rho(\tilde{t})$ is called a stability radius of the line balance b_0.

Let $W(b_r, t)$ denote the set of subsets $\widetilde{V}_k^{b_r}$ defined in (1), $k \in \{1, 2, \ldots, m\}$, for which $t(V_k^{b_r}) = c(b_r, t)$. The following sufficient and necessary condition for a zero value of the stability radius has