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Two-product storage-capacitated inventory systems: A technical note

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ABSTRACT

This paper considers a two-product inventory model with limited storage capacity and constant demand rates. We aim at finding an ordering policy that minimizes the cost per time unit. In the literature, several solution methods have been developed for this problem, but these are limited to very restrictive classes of policies. We consider a much more general class where the order quantity of one of the products is allowed to vary. These policies are still cyclic and easy to implement. Closed-form expressions are derived for determining the optimal order quantities. It is shown that savings of up to 25% are possible compared to existing approaches.

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1. Introduction

The majority of inventory control literature is based on cost minimization and ignores capacity constraints. In real life, such constraints often do exist, either storage or production related. This paper focuses on storage related capacity restrictions, which may result from physical limitations or internal budget restrictions.

Many textbooks, including Hadley and Whitin (1963), suggest coping with storage capacity limitations through partitioning of the storage capacity. In this approach, each product gets its own share of the storage capacity. This approach is mainly referred to as the *Lagrange multiplier approach*, but the terms *independent cycle*, *independent solutions* and *fixed storage* approach are also used. The advantage of this approach is that the negative effects of the capacity restriction can be spread evenly over the products. An important disadvantage is that the capacity is not used efficiently. Indeed, for deterministic demand, the average capacity usage is always 50%.

Another approach is to use the same cycle time for all products. This approach is referred to as the *fixed cycle*, *pure cycle*, *rotation cycle* or *common cycle* approach. In this approach, a common cycle time is determined and all orders are phased within this cycle, such that the storage capacity is used more efficiently. The main problem is to determine this phasing, i.e. the sequencing of the products, usually called the *staggering* of the products. Homer (1966) was the first who solved this staggering problem to optimality. His result was rediscovered, independently, by Page and Paul (1976), Zoller (1977), Hall (1988). The main advantage of this approach is that capacity is used more efficiently. An important disadvantage, however, is that forcing ordering cycles to become equal can be very costly.

Combining the two main approaches has also been suggested. Page and Paul (1976) provide a method that clusters the products. All products within a cluster have the same order interval, leading to an efficient use of the capacity, while the order intervals vary across the clusters. Anily (1991) provides another method that builds on the same rationale but determines the clusters in a different way. She shows that the performance of her clustering method does not exceed some lower bound on the costs by a factor larger than $\sqrt{2}$.

Besides combinations of the two main approaches, several authors generalized the methods. Gallego et al. (1996) generalized the fixed cycle approach to a powers-of-two policy. Order quantities remain fixed. The authors provide heuristics that determine ordering policies. An even more general approach is used by Hartley and Thomas (1982). They consider a two-product model in which they allow each product to be ordered several times per cycle, though still in fixed amounts. They provide an optimal solution procedure in a companion paper (Thomas and Hartley, 1983). Murthy et al. (2003) and Boctor (2010) consider the problem of offsetting the replenishment cycles by integer multiples of some base period, and use the result that, if the integer multiples of two items are not relatively prime, it is possible to offset their cycles such that the peaks of their inventory cycles never coincide over an infinite time horizon. Murthy et al. propose a heuristic for this framework, which is improved by Boctor.

Several authors provide structural insights. Anily (1991) provides a lower bound on the costs of inventory policies with constant order quantities and Gallego et al. (1996) prove that this bound holds even for varying quantities. Finally, Gallego et al. (1992) prove that the problem of determining an optimal ordering policy subject to a storage capacity restriction is strongly NP-complete.

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All the above discussed contributions consider policies with constant order quantities. Some other contributions do focus on varying order quantities. Hariga and Jackson (1995) formulate a nonlinear program that minimizes holding and ordering costs by selecting order quantities and the overall cycle length. They provide a heuristic solution procedure and conditions under which fixed order quantities are optimal. However, throughout the entire paper, they assume that the sequence of orders is given, which, in fact, constitutes the most challenging part of the determination of an optimal ordering policy.

Güder et al. (1995) also allow varying order quantities. They provide a myopic procedure that determines order quantities one by one, which in general results in a non-cyclic ordering policy. The order quantities that the procedure selects are based on the Economic Order Quantity as well as the available storage space. An upper bound on the optimality gap is not provided.

In this paper, we present an exact approach for the two product inventory model with a capacity constraint and varying order quantities. We provide closed-form expressions for optimizing order quantities and ordering moments for a very general class of ordering policies. We further derive theoretical and numerical results on the suboptimality of existing approaches, which turns out to be considerable in many situations.

This paper proceeds as follows. Section 2 describes the inventory system and introduces the concepts of simple and general cycles. In Section 3, structural properties of the optimal simple cycle policy are derived and used to determine the optimal timing and quantity of orders. Section 4 compares the cost performance of the optimal simple cycle policy to that of existing approaches. Section 5 shortly discusses general cycle policies. We end in Section 6 with a summary of the key findings and insights, and a discussion of research opportunities.

2. System description

Consider a two product, infinite time horizon inventory system. The products share a common storage resource with limited capacity. The products are numbered 1 and 2. Demand rates are deterministic and denoted by d_1 and d_2. Demand is measured in *capacity units per time unit*, i.e. as the rate with which capacity decreases, to allow for normalization of the capacity to unity. Subscripts refer to the product number. Lead times are constant and backorders are not allowed.

The objective is minimizing costs per time unit by determining ordering *moments* and *quantities*. This average cost per time unit is denoted by C and based on a cost per order for each of the two products, which are denoted by A_1 and A_2, respectively. We will focus purely on the ordering costs as we consider situations where ordering quantities are restricted by the limited available capacity rather than the need to avoid excessive holding costs.

Our attention is restricted to cyclic solutions. A *cyclic solution* is a solution in which ordering moments and quantities are described for a finite time interval, the *cycle*, and the inventory levels at the beginning and at the end of this cycle are equal. As a result, the ordering policy within the cycle can be repeated infinitely many times. The length of the cycle is denoted by T, which is a decision variable that will vary for different configurations. Cycles in which at least one of the products is ordered only once, which implies that its order quantity does not vary, will be called *simple cycles*. Cycles without any further restrictions will be referred to as *general cycles*.

3. Simple cycles

Let the *base product* refer to the product that is ordered once per cycle. If both products are ordered once, an arbitrary product can be selected as the base product. We will present the analysis for the case where product 1 serves as the basis, leading to the optimal policy of that type, and than 'copy' the result for the other case where product 2 is the base product.

The replenishment order of product 1 is referred to as the *product 1 order* and its order quantity is denoted by Q_1. Recall from Section 1 that the capacity is normalized to one and hence $Q_1 \le 1$. The number of orders of product 2 during the cycle is denoted by m, which is restricted to be integer. The product 2 orders are numbered according to their position in the cycle, starting from the product 1 order. Hence the i-th product 2 order after the product 1 order will be named *product 2 order* i and its quantity will be denoted by $Q_{2,i}$, for $i = 1, \ldots, m$, where $0 < Q_{2,i} \le 1$.

The above properties and notations lead to the following expression for the cycle time T of simple cycles:

$$T = \frac{Q_1}{d_1} = \frac{\sum_{i=1}^{m} Q_{2,i}}{d_2}. \tag{1}$$

The cost per time unit, which should be minimized, is

$$C = \frac{A_1 + mA_2}{T}. \tag{2}$$

It is easy to see that, for fixed m, maximization of T implies minimization of C. In turn, (1) shows that maximizing the order quantities maximizes T. This observation suggests that (a) the products should be ordered only when their stock is empty, and (b) the order quantities fill all remaining capacity, i.e. order quantities are of maximum size. In the next subsection, we show that optimal solutions indeed always satisfy properties (a) and (b) and we derive expressions for optimal order quantities by applying these.

3.1. Deriving optimal order quantities

Let us assume that we have decided on the base product (numbered 1) and the number, m, of orders per cycle for product 2. In the next subsection, we will show how to select the base product and the value of m optimally, in order to find the overall best simple cycle policy.

The following theorem states that all orders must be of maximum size. We remark that Hariga and Jackson (1995) obtain the same results under the objective of minimizing the storage capacity, but not under cost minimization as considered here.

Theorem 1. *Under an optimal simple cycle policy, every order is of maximum size, i.e. uses up all spare storage capacity when it arrives.*

A proof by contradiction is provided in Appendix A.

The following corollary shows that next to being of maximum size, orders should always arrive just in time. A formal proof is provided in Appendix A, but the logic is as follows. An order that arrives early (when the inventory level is still positive) can be postponed, which leads to an alternative policy with an order of non-maximum size. As the alternative policy cannot be optimal (using Theorem 1), neither can the original policy. We remark that Anily (1991) proves the same, but along different lines.

Corollary 1. *Under an optimal policy, a product arrives exactly when its inventory level reaches zero.*

Now, we derive optimal order quantities as follows. Since the product 1 order is of maximum size, by Theorem 1, and arrives when the product 1 stock level is zero, by Corollary 1, the stock level of product 2 at that time is $(1 - Q_1)$. So, product 2 order 1 will be placed $\frac{1-Q_1}{d_2}$ time units later and, by Theorem 1, its quantity is equal to the total demand over this period, i.e.

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