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A coordinated manufacturer-retailer model under stochastic demand and production rate

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ABSTRACT

This paper deals with a manufacturer-retailer model for a two-stage supply chain. The retailer and the manufacturer face random demand and random yield, respectively. Each time the manufacturer switches from an idle mode to a production mode a fixed cost is incurred. Inventories can be deployed both at the manufacturer and the retailer. This description of a two stage system is more detailed than generally seen in the literature. We first develop a Markov chain model to compute the optimal coordinated decision policy. This model serves as benchmark to study the performances of three non-coordinated models. In all three models the retailer first computes an optimal (s,S) policy which the manufacturer then comply with. The difference among three models is how the backlogs at the manufacturer are handled. The numerical results show that the non-coordinated model in which the backlog is immediately canceled generally performs best.

1. Introduction

Companies are realizing the necessity of a more efficient management of inventories across the supply chain through better coordination. The reason is that the products can be provided at a reduced cost in coordinated supply chains. Traditionally, the production and inventory policies for the manufacturer and the retailer were managed independently. This independent decision behavior usually cannot assure that the supply chain members as a whole reach the optimal state. To overcome this difficulty, the coordinated vendor-buyer model is developed, where the joint total relevant cost for two members is minimized. In the literature, one stream of research deals with the coordinated vendor-buyer problem referred to as integrated production-inventory and joint economic lot sizing (JELS) problem. This paper is related to this stream of research.

The joint optimization concept for buyer and vendor was first described by Goyal (1976). He developed an integrated vendor-buyer model under the assumption of infinite production rate for the vendor and lot-for-lot policy for the shipments from the vendor to the buyer. Bannerjee (1986) considered the vendor manufacturing for stock at a finite rate and delivering the whole batch to the buyer as a single shipment. Goyal (1988) generalized that model by allowing a production batch to be split. Lu (1995) set out the optimal production and shipment policy when all shipment sizes are equal. Later, Goyal (1995) developed a model where the shipment size increases by a factor. Hill (1997) took this idea one step further by considering the geometric growth factor as a decision variable. Hill (1999) showed that the structure of optimal policy includes shipments increasing in size according to a geometric series followed by equal-sized shipments.

Many works have been devoted to developing integrated production-inventory models under a variety of circumstances. For instance, Huang (2004) studied the effect of quality on lot size. Kim et al. (2006) studied joint procurement-production-delivery policy in a single-manufacturer, multiple-retailer supply chain. Later, Abdul-Jalbar et al. (2007) developed a model for a single-vendor two-buyer problem. Siajadi et al. (2006) developed the multi-buyer single-vendor model. Zavanella and Zanoni (2009) developed an integrated single-vendor multi-buyer model assuming a shared management of the buyers' inventory.

The existing literature encompasses a number of different categories (see e.g., Rau and OuYang, 2008; Yang et al., 2008; Sajadieh and Jokar, 2009; Sajadieh et al., 2010; Sari et al., 2012; Sajadieh et al., 2013; Sajadieh and Thorstenson, 2014; Wang et al., 2015; Glock and Kim, 2015). We refer to Ben-Daya et al. (2008) and Glock (2012) for a comprehensive review of the problems. It should be pointed out that although there are considerable studies covering the different dimensions of the problem; most of them are constrained to deterministic conditions, which limit their applicability.

There is also a stream of literature known as multi-echelon inventory systems where a number of installations are coupled to

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each other. One critical objection is that the underlying supply process is seldom described. Most often it is assumed that the capacity is infinite through a fixed lead time, though exceptions; Song and Zipkin (1996). We refer to Axsäter (2006) and Zipkin (2000) for more details.

Another stream of research is single-stage production-inventory models: Gaver (1961), Gavish and Graves (1980), de Kok (1985), and de Kok (1987). A general characteristic for these contributions is that they focus on (m,M) policies, only allowing one or two positive (and predetermined) production rates to be used. For new references see Wang et al. (2002).

Given that the real business environment is usually stochastic it will be very relevant to study the integrated models in non-deterministic situations. It is our motivation in this paper to develop an integrated production-inventory system. We abandon the assumption of deterministic demand and production rate in the hitherto existing joint economic lot sizing models, and analyze the problem where these two essential parameters are stochastic.

The present paper deals with the optimal production and inventory policies of a two-stage supply chain under variability such as demand and production rate. The contribution of paper is to develop an integrated production-inventory model as well as three non-coordinated models for the circumstances in which the demand and the production rates are both stochastic. We first obtain the optimal policies that minimize the expected total cost per period for each model and then compare the non-coordinated cases. For each policy we formulate the problem as a discrete Markov Decision Process (MDP). We also analyze how the coordination between two supply chain members is affected when there are uncertainties with respect to the production rate as well as the end customer demand. The numerical results show that the non-coordinated model in which the backlog is immediately canceled (i.e., the retailer and manufacturer interacts under a lost-sales assumption) generally performs best, and the increase in the total cost compared to the ideal (but maybe impractical) coordinated model is moderate.

The paper is organized as follows. In Section 2, the assumptions and notation are introduced. Section 3 discusses the coordinated policy for the manufacturer and the retailer as well as the non-coordinated models. The solution approaches employed are also discussed in this section. Section 4 presents numerical examples and sensitivity analysis. Finally, the main findings and further research directions are summarized in Section 5.

2. Problem assumptions and notation

The mathematical models are based on the following assumptions:

1. The model deals with a single manufacturer and single retailer for a single product.
2. In each period the retailer faces a stochastic demand D which follows a discrete distribution. The random variable D can only attain the integer values in the interval $[d_{\min}, d_{\max}]$, where both integers $d_{\min}$ and $d_{\max}$ are non-negative, and $d_{\min} \le d_{\max}$. The demands in subsequent periods are assumed to be independent.
3. A finite production rate P for the manufacturer is considered which follows a discrete distribution. The random variable P can only attain the integer values in the interval $[p_{\min}, p_{\max}]$, where both $p_{\min}$ and $p_{\max}$ are positive, and $p_{\min} \le p_{\max}$. The production rates in subsequent periods are assumed to be independent.
4. The average production rate is greater than the average demand rate, i.e., $E[P] > E[D]$. The production and the demand rates are assumed to be independent.
5. Transportation time to replenish the retailer's order, L, is constant.
6. Shortages at the retailer are backordered.
7. Inventory and backorder costs are charged based on the items available at the end of period.
8. The time horizon is infinite.

The cost parameters are:

K	Set-up cost at the manufacturer.
h^M	Unit inventory cost at the manufacturer.
A	Fixed cost for making transhipment from the manufacturer to the retailer.
h^R	Unit inventory cost at the retailer.
b^R	Unit backorder cost at the retailer.

The mathematical notation is straightforward. For any real number f: $f^+ = \max(f, 0)$. For any logical expression Z, $I_Z = 1$ if Z is true, and 0 otherwise. For any real number x, the symbols $\lceil x \rceil$ and $\lfloor x \rfloor$ are used for rounding x respectively up and down to its nearest integer. For any positive integer T, the random variable $D(T) = \sum_{n=1}^{T} D_n$is the total demand observed in T subsequent time periods where the random variables D_n $(n=1,..., T)$ are independent and identically distributed.

3. Mathematical models

In this section, we derive the optimal policy of the coordinated system. For comparative purposes, however we obtain the retailer and manufacturer policies, if each party minimizes its costs independently. The policies and costs are then compared to the case of coordinated system. We formulate the systems using the discrete time Markov Decision Process. The MDP is a class of stochastic sequential processes in which the expected cost until next decision epoch and the transition probability depend only on the current state of the system and the current action. Each model consists of states, actions, penalties and transition probability.

3.1. Coordinated model

3.1.1. Mathematical model

Suppose that both parties decide to cooperate and agree to share information to deal with the uncertainty of demand and supply. The state space is three dimensional, denoted by (y, i, j). The binary variable y is 1 if the manufacturer was in production mode in the previous period, and 0 otherwise. The integer i represents the on-hand inventory at the manufacturer. It attains the values 0,1, …, $i_{\max}$ where $i_{\max}$ is a upper bound on the inventory. As there exists an upper bound on the inventory, the excess finished items are thrown away at a unit cost π^{Disp}. It does not mean that we really dispose the finished items of good quality. Setting $i_{\max}$ sufficiently high ensures that it happens with a probability almost zero, i.e. disposing the finished items of good quality will not happen. Thus, in reality we have an infinite state space with respect to the variable i. The integer j represents the net inventory at the retailer (on-hand – backorders)+the amount of goods in transit. It is not the commonly known inventory position as any outstanding orders at the manufacturer are not included in j, rather it is the realized inventory position as it is coined by Axsäter (2006). We assume that j can attain all integer values in the interval $[j_{\min}, j_{\max}]$ where the integer $j_{\min}$ is non-positive. For instance Tijms (1994) proposes that $j_{\min}=0$ in his (s, S) model.

If the manufacturer cannot have inventory above $j_{\min}$ at the start of the next period, he must expedite the needed materials