Int. J. Production Economics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Production economics and the learning curve: A meta-analysis

Eric H. Grosse*, Christoph H. Glock, Sebastian Müller

Institute of Production and Supply Chain Management, Technische Universität Darmstadt, Hochschulstr. 1, 64289 Darmstadt, Germany

A R T I C L E I N F O

Article history:
Received 13 April 2014
Accepted 18 June 2015

Keywords:
Learning
Learning curve
Experience curve
Curve fitting
Literature survey
Literature review
Meta-analysis

A B S T R A C T

For almost a century, researchers and practitioners have studied learning curves in production economics. Learning, in this context, refers to performance improvements of individuals, groups or organizations over time as a result of accumulated experience. Various learning curves, which model this phenomenon, have been developed and applied in the area of production economics in the past. When developing planning models in production economics, the question arises which learning curve should be used to best describe the learning process. In the past, the focus of the literature has been on empirical studies that investigated learning processes in laboratory settings or in practice, but no effort has been undertaken so far to compare existing learning curves on a large set of empirical data to assess which learning curve should be used for which application. This study systematically collected empirical data on learning curves, which resulted in a large database of empirical data on learning. First, the data contained in the database is categorized with the help of meta-tags along different characteristics of the studies the data was taken from. Second, a selection of well-known learning curves is fitted to the empirical datasets and analyzed with regard to goodness of fit and data characteristics. We identify a set of data/task characteristics that are important for selecting an appropriate learning curve. The results of the paper may be used in production economics to assist researchers to select the right learning curve for their modeling efforts.

1. Introduction

Since Wright's (1936) seminal work on the functional relationship between the time required to perform a task and task repetition, a plethora of works has been published that investigated this functional relationship that is also termed the learning curve. Learning (or experience) curves assume that performance (output) improves as a task is repetitively performed, which is attributed to experience that is accumulated by the individual or group performing the task. Learning curves have frequently been the subject of research. Empirical studies focused on measuring learning by collecting empirical data, either in laboratory settings or in field studies. Learning effects were observed in various areas, such as assembly production (Shafer et al., 2001; Smunt and Watts, 2003), online ordering in supply chains (Kull et al., 2007), manual order picking (Grosse and Glock, 2013), or construction (Hinze and Olbina, 2009), to name just a few examples. Although the concept of learning curves in the field of production economics has been introduced almost a century ago, it is still of importance for manufacturing firms, for example as a performance measure, an aid in setting labor standards, a forecasting tool, or an application in decision support tools. Recent examples, for instance the market launch of Boeing's Dreamliner, confirm the practical importance of learning curves (Nolan, 2012).

Learning curves can be of multivariate or univariate type, where log-linear, exponential and hyperbolic models have most often been used (Anzanello and Fogliatto, 2011). Besides studying learning empirically, many authors have modeled the effects of learning on industrial and logistics processes by including learning curves in decision support models. Examples are inventory models that consider learning in the production rate, in setups or in fuzziness (e.g., Jaber et al., 2008, 2009; Kazemi et al., 2015), supplier selection models (e.g., Glock, 2012), models of manual order picking that consider picker learning (e.g., Grosse et al., 2013; Grosse and Glock, 2014), or vehicle routing models that involve driver learning (e.g., Zhong et al., 2007). Learning curves and their applications have been surveyed in a number of literature reviews, such as in Yelle (1979), Anzanello and Fogliatto (2011), or Fogliatto and Anzanello (2011).

* Corresponding author. Tel.: +49 6151 165281.
E-mail addresses: grosse@pscm.tu-darmstadt.de (E.H. Grosse), glock@pscm.tu-darmstadt.de (C.H. Glock), sebastianmueller-lm@gmx.de (S. Müller).

http://dx.doi.org/10.1016/j.ijpe.2015.06.021
0925-5273/

Surprisingly, the question of how different learning curves perform and which learning curve to use in which application has not yet been addressed in a comprehensive study. Researchers and practitioners alike face the problem of selecting an appropriate learning curve each time learning effects are modeled, which can be challenging and time-consuming given the large number of learning curves that have been developed in the past. To assist researchers and practitioners in their efforts to model human learning, this paper provides a comprehensive study of learning curves and their applicability. Based on an extensive review of the literature, empirical data on learning is collected, which is then used to evaluate a selection of popular learning curves. With the help of meta-tags (see Section 3.4 for a detailed description and definition of meta-tags) on the general setup and purpose of the datasets contained in our sample, we compare the performance of different learning curves and derive propositions as to which learning curves perform best in which application. The results of this paper may assist researchers and practitioners to select learning curves for future studies.

The remainder of this paper is structured as follows. Section 2 first discusses popular learning curve models. The results of a comprehensive literature review on empirical studies of learning are presented in Section 3. Section 4 analyses the goodness of fit of the learning curves presented in Section 2 on the empirical datasets obtained in Section 3. Section 5 summarizes the findings of the study and concludes the paper.

2. Learning curve models

This section presents a selection of learning curves that have frequently been studied in the past. Learning curves presented below have been selected based on their popularity, which was evaluated with the help of the reviews cited above, and to make sure that a broad range of learning curves is used for data fitting. We note that the literature contains many more models of learning that are not discussed in this paper, and refer the reader to the reviews that were cited above.

2.1. Log-linear models

2.1.1. Wright's model (WLC)

A seminal paper on learning curves is the one of Wright (1936), who showed that the average unit production costs in airplane assembly reduced as a function of the number of airplanes produced. He suggested that this phenomenon is caused by increasing worker skill levels, fewer setups and a decreasing number of errors. Wright's learning curve has the following form:

$$y_x = y_1 \cdot x^{-b}, \tag{1}$$

where y_x is the time needed for the xth repetition of the task, y_1 is the time required for the first repetition, x the number of repetitions, and b the slope of the learning curve (learning exponent), with $0 < b < 1$. Note that Wright's learning curve (and other log-linear models) can be used to model both reductions in time or in cost as a result of learning.

2.1.2. Plateau model (PM)

The Plateau model is similar to the one proposed by Wright, with the difference that a constant C is added to the model to take into account that a minimum time exists for performing a task that is independent of the learning effect (Baloff, 1971). The plateau learning curve is formulated as

$$y_x = C + y_1 \cdot x^{-b} \tag{2}$$

2.1.3. Stanford B model (SBM)

The Stanford B learning curve extends Wright's learning curve by considering prior experience (Carlson, 1973). The model assumes that an equivalent of $B > 0$ cycles has been processed earlier, either because the same or a similar task has been performed, which led to the acquisition of knowledge. The Stanford B model is formulated as follows:

$$y_x = y_1 \cdot (x + B)^{-b} \tag{3}$$

2.1.4. De Jong's model (DJM)

De Jong (1957) assumed that there is an incompressible component in each process where no learning and thus no productivity improvement occurs, and thus extended Wright's (1936) learning curve by adding a factor of incompressibility to the model. De Jong's learning curve has the following form:

$$y_x = y_1 \cdot (M + (1 - M) \cdot x^{-b}) \tag{4}$$

The factor M ($1 \geq M \geq 0$) depends, for example, on the degree of automatization of the production process. If a production process is partially automatized, we may assume that no learning takes place in automated tasks. Thus, the fewer manual tasks a production process contains, the earlier learning may be assumed to plateau, which is expressed by a higher value for M.

2.1.5. S-curve model (SCM)

The S-curve model combines the characteristics of the Stanford B model and De Jong's model. The name derives from the fact that this learning curve is s-shaped when plotted in logarithmic scale. It can be expressed as follows (Nembhard and Uzumeri, 2000):

$$y_x = y_1 \cdot (M + (1 - M) \cdot (x + B)^{-b}) \tag{5}$$

2.1.6. Jaber–Glock learning curve model (JGLCM)

The *JGLCM* extends the dual-phase learning curve introduced by Dar-El et al. (1995) and accounts for the fact that in most industrial tasks, both cognitive and motor learning occur (Jaber and Glock, 2013). The *JGLCM* consists of two components, cognitive and motor, where p represents the share of both types of learning. It is modeled as follows:

$$y_x = p \cdot y_1 \cdot x^{-b_c} + (1 - p) \cdot y_1 \cdot x^{-b_m}, \tag{6}$$

where b_c is the learning exponent for cognitive learning and b_m the one for motor learning.

2.2. Exponential models

Exponential learning curve models contain more parameters than log-linear models to account for empirically observed characteristics (such as worker's prior experience) and to include more information on the learning process. Exponential models that are fitted to empirical data in this paper are discussed briefly in this section.

2.2.1. 2-Parameter exponential model (2PE)

The 2-parameter exponential model of Mazur and Hastie (1978) is formulated as

$$y = k \cdot \left(1 - e^{-(t/R)}\right), \tag{7}$$

where y represents the number of units produced since the start of production, t the time that has elapsed since the start of production (or the time that has elapsed during training), k the prediction of maximum performance after an infinite amount of training ($k \geq 0$), and R the learning rate parameter which measures how fast an individual learns.