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Fill rate in a periodic review order-up-to policy under auto-correlated normally distributed, possibly negative, demand

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ABSTRACT

We investigate the inventory service metric known as the fill rate—the proportion of demand that is immediately fulfilled from inventory. The task of finding analytical solutions for general cases is complicated by a range of factors including; correlation in demand, double counting of backlogs, and proper treatment of negative demand. In the literature, two approximate approaches are often proposed. Our contribution is to present a new fill rate measure for normally distributed, auto-correlated, and possibly negative demand. We treat negative demand as returns. Our approach also accounts for accumulated backlogs. The problem reduces to identifying the minimum of correlated normally distributed bivariate random variables. There exists an exact solution, but it has no closed form. However, the solution is amenable to numerical techniques, and we present a custom Microsoft Excel function for practical use. Numerical investigations reveal that the new fill rate is more robust than previous measures. Existing fill rate measures are likely to cause excessive inventory investment, especially when fill rate targets are modest, a strongly positive or negative autocorrelation in demand is present, or negative demands exist. Our fill rate calculation ensures that the target fill rate is achieved without excessive inventory investments.

1. Introduction

The fill rate is a popular measure of inventory service in high volume industries as it directly measures the customer's experience of demand fulfilment. The fill rate is defined as *the proportion of demand fulfilled directly from inventory* (Silver et al., 1998, p. 245; Sobel, 2004; Axsäter, 2000, p. 57). However, this simple definition hides technical details that are often overlooked. In particular there are issues with double counting of backlogs, lead times, autocorrelation in demand, cross-correlation between net stock and demand, negative demand, and the distribution of demand and net stock. This paper presents a procedure for identifying the true fill rate obtained in the presence of these complicating factors.

1.1. Contribution

Our contribution is the exact expression for the long run fill rate under auto-correlated, possibly negative demand. It is important to have an exact expression as errors can cause excessive inventory investments or over-optimistic fill rate guidance. Indeed, when demand is negatively or strongly positively auto-correlated excessive fill rates are achieved indicating that an opportunity to reduce safety stocks exists. We extend the definition of the fill rate to be compatible with negative demand. It is a generalisation of the common fill rate definition and will produce identical results for non-negative demand.

Existing fill rate measures provide nonsensical results in the presence of negative demand—either fill rates of over 100% or below 0%. Additionally, simulation results can differ significantly from theoretical guidance. Our proposed approach is mathematically correct and numerically accurate, and gives logical and consistent results. The solution reduces to the identification of the distribution of the minimum of two normally distributed correlated random variables. This distribution has an exact solution, but no closed form solution exists. However the problem is amenable to numerical methods. For practical work we provide an Excel Add-in for calculating the true fill rate. We highlight the research gaps and our contribution to the field in Fig. 1.

1.2. Motivation

Demand patterns can be both auto-correlated and possibly negative. For example, Fig. 2 illustrates a consumer electronics

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0925-5273/

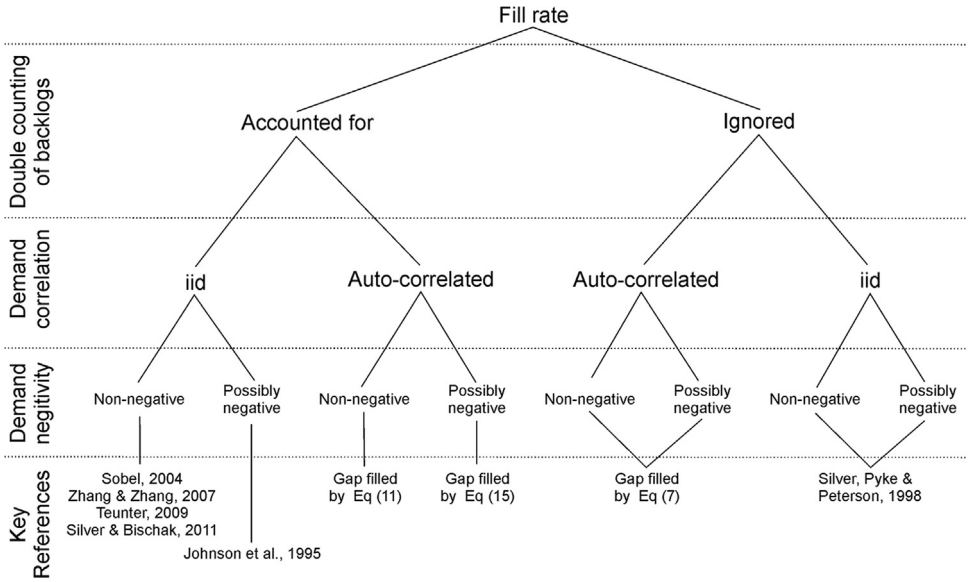

Fig. 1. Our contribution to the fill rate literature.

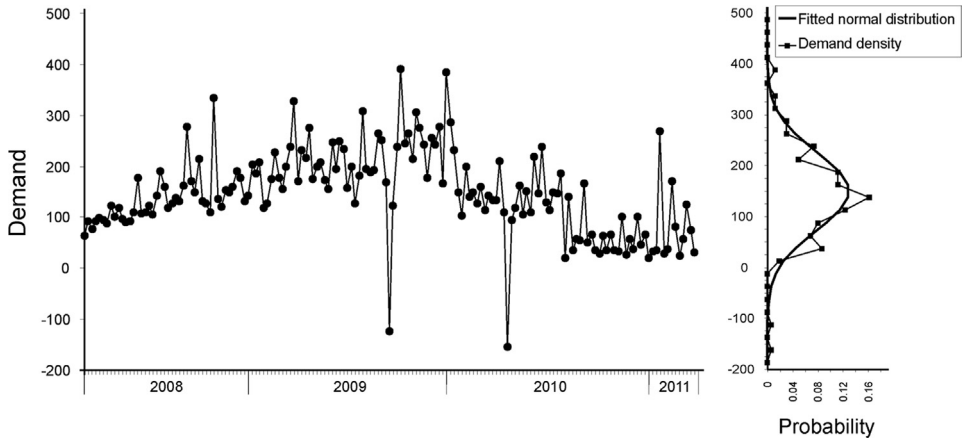

Fig. 2. A real-life demand pattern with returns from the consumer electronics industry.

product with a demand that is approximately normally distributed but is not independent and identically distributed (i.i.d.) as there are clear rising and falling trends. It has weekly demand with a mean demand of 146.6 and a standard deviation of 82.7. It also contains two negative demands. Negative demand in a period indicates that the returns from customers are larger than those delivered. The fitted normal distribution in the density plot has a mean of 150.2 and a standard deviation of 76.7. This was determined by minimising the squared error in the density plot after removing the two outliers that were more than three standard deviations from the mean.

Returns can be significant, particularly in industries such as books, consumer electronics and fashion retailing. We have also noticed that when a large batch of raw materials is checked out of stores and only partially used in production during a period, the remaining raw materials can be returned to the stores in a following period. This procedure can result in a negative demand being recorded in the latter period. Stock adjustments to correct accumulated recording errors can also result in negative demand. Johnson et al. (1995) provide further justification for negative demands.

Practical fill rate targets are most likely to be above 50%. However, it is mathematically plausible fill rate targets could be anywhere between 0% and 100%. Sapra et al. (2010) discuss the *inventory withholding* strategies of fashion and luxury manufacturers and retailers. This sector has experimented with limiting supply and creating waiting lists to generate a sense of scarcity and exclusivity that may over time increase both the demand volume and the sale price that can be commanded. Here low fill rates are purposely targeted.

1.3. Summary of results

We explore the fill rate in a setting with normally distributed, auto-regressive moving average demands (Box and Jenkins, 1976). We assume that inventory is managed by a linear, discrete time, order-up-to (OUT) replenishment policy and that lead times are arbitrary but constant. We develop our measure analytically and verify its performance via simulation. This reveals that our fill rate is more robust than previous ones, giving accurate predictions over the whole range of fill rates, for any proportion of negative demand, for both i.i.d. and auto-correlated demands. Numerical investigations reveal that our approach is particularly useful when the probability of negative demands is large and fill rates near 100% or 0% are required.

1.4. Structure of the paper

The structure of this paper is as follows. Section 2 reviews the literature and highlights the research gap. Section 3 reviews background knowledge of the normal distribution and the distribution