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A note on the rationing policies of multiple demand classes with lost sales

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ABSTRACT

We study inventory rationing in a system with multiple demand classes and lost sales. It is assumed to have at most one outstanding order, resulting in two periods in an order cycle separated by the time of order release. We review the most related work by Melchiors (2001, 2003) (Ph.D. thesis, University of Aarhus, Int. J. Prod. Econ. 81–82, (11), 461–468), and find that the existing approximated and optimal policies are not easy to obtain due to computational complexity. Also as the rationing issue before order release is not well addressed in literature, in this paper we prove the static rationing being optimal. Furthermore in such a system with two distinct periods, the optimal rationing policy is a combination of a dynamic policy during the replenishment lead time and a static policy before order release. In order to make the rationing policies to be readily used in practice, we introduce two approximated methods for calculating the rationing levels in two periods, respectively. The results, in particular the combination of static and dynamic rationing, outperform the existing approximations in literature. In addition, the computation is obviously simplified due to the efficient algorithm of dynamic rationing and the explicit expressions of static rationing.

1. Introduction

The concept of multiple demand classes is to remove the traditional assumption of homogeneous customers but consider various characteristics of different customers. Customers may differ in the aspects such as demand quantity, price, delivery and service requirement. Inventory rationing is an approach to provide distinct services to different customers from a single inventory system while it still keeps the economic scale of the operation. For example, reserving a certain amount of inventory for customers with higher backorder costs will prioritise their demand fulfilment and thus improve the overall economic performance in the system. The study of inventory rationing first stems from the inventory management in service parts, where demand comes from both normal replenishments and emergency orders. Nowadays, inventory rationing can be applied in many fields such as manufacturing, service and retailing to provide premium customers a better service.

We study an inventory rationing problem in a (s, Q) system with multiple demand classes. Unsatisfied demand is assumed to be lost. This assumption reflects the unwillingness of customers to wait for replenishment. Lost-sale is considered to be a major consequence in many practical settings when stockout occurs (Gruen et al., 2000). We further assume that there is at most one outstanding order. This assumption mainly helps bring analytical results for the model. It is also consistent with some practical situations where an ordering cost is relatively high and a lead time is shorter than an order cycle. For example, inventory systems for products that are transported by air usually have a short lead time but a high ordering cost. As a result, there is often a large difference between order quantity and reorder point, and the inventory system seldom has more than one outstanding order.

The above assumption suggests two periods in one order cycle, separated by the time of order release. In other words, an order cycle consists of two periods: one before order release without outstanding order, and one after order release with an outstanding order, i.e., the replenishment lead time. The assumption is realistic for many real cases when order cycle is long and replenishment lead times are not overlapped. Since the length of the first period is uncertain due to the stochastic demand and the length of the second period is constant, they may require distinct rationing policies for two periods rather than one single policy.

The rationing problem in such a system is mainly studied by Melchiors et al. (2000, 2001, 2003). Melchiors et al. (2000) propose a static rationing policy which employs constant rationing

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levels for the whole order cycle. Melchiors (2001) further proves that the optimal rationing policy in the constant lead time period is a dynamic policy. He develops a policy iteration method to obtain the optimal dynamic policy. As the iteration method is difficult to calculate, Melchiors (2003) then introduces an RTR (restricted time-remembering) policy as an approximation for easy implementation. Based on our study on the literature, we find that the rationing before order release has not been addressed properly and the existing approximated policies for both periods are time-consuming for computation. Hence, in this note we investigate structural properties of such a system with two distinct periods and develop simple and efficient approximations of rationing policies in both periods.

The problem of inventory rationing among multiple demand classes was first studied by Veinott (1965). Teunter and Haneveld (2008) present a thorough review on the literature over the past several decades. We hence give a brief review on the latest articles which are most related to our study.

Inventory rationing policies have to be embedded in inventory systems with various ordering policies. In continuous review systems, (s,Q) (or sometime called (r,Q)) policy gains the most attentions. Nahmias and Demmy (1981) develop the expressions for describing backorders and service levels for a (s,Q) system with at most one outstanding order. However, it is difficult to obtain the optimal rationing level along with s and Q, especially when multiple outstanding orders exist. To tackle this problem, Deshpande et al. (2003) design a threshold clearing mechanism, which leads to closed-form formulae for performance measures and near optimal solutions. Wang et al. (2013) adopt this threshold clearing mechanism and study the rationing policies in a (s,Q) system with a mixture of service criteria. Fadıloğlu and Bulut (2010) propose an alternative method to solve the problem with multiple outstanding orders. They adjust the inventory level by including outstanding orders (represented by an exponential function). By this means, the optimal rationing levels can still be set as static rather dynamic. Hung and Hsiao (2013) apply inventory rationing in a warehouse and develop two approximated rationing policies in a (s,S) system. Chen et al. (2012) discuss the application of similar inventory rationing systems in group buying.

In periodic review systems, Hung et al. (2012) develop a heuristic to compute dynamic rationing levels in a single period model and a multi-period model. Chew et al. (2013) investigate the application of dynamic rationing in a multi-period inventory problem. By discretising the time period into small intervals, they formulate the problem as a Markov decision model and characterise the optimal solutions. Wang and Tang (2014) further generalise the model by considering a mixture of backorder and lost-sale type demand classes. Liu et al. (2015) obtain closed form expressions for dynamic rationing levels based on certainty equivalence principle in a single period system. According to our literature review and the review in Teunter and Haneveld (2008), the studies considering lost sales and applying dynamic rationing are very limited in literature.

Our paper focuses on the theoretical research on rationing in a (s,Q) system while assuming lost sales, with the most relevant studies of Melchiors (2001, 2003). In the next section, we reformulate the exact model for the system and introduce the existing approximated policies of Melchiors (2001, 2003). In Section 3, we propose a dynamic rationing policy for the period of lead time. In Section 4, we propose a simple static rationing level for the period before order release. Conclusions are drawn in Section 5.

2. Models and policies

We study a continuous review (s,Q) inventory system with J demand classes indexed by j. The demand of each class is Poisson distributed with demand rate λ_j. Any unsatisfied demand is lost and incurs a penalty cost π_j. Without loss of generality, we rank the classes such that $\pi_1 > \pi_2 > \ldots > \pi_J > 0$. For later use, let $\Lambda_i = \sum_{j=1}^{i} \lambda_j$ be the aggregated demand rate from demand class 1 to class i, and $\Pi_i = \sum_{j=i}^{J} \lambda_j \pi_j$ the aggregated weighted penalty cost from class i to class J. Define $p(x,\mu)$ and $P(x,\mu)$ as the probability mass function and the cumulative probability function of Poisson distribution with mean value μ.

Assume that the order quantity Q is always larger than the reorder point s. In a lost sales system, it implies there is at most one outstanding order. Although considering multiple outstanding orders apparently covers a more general situation (Fadıloğlu and Bulut, 2010), the overlaps of the order cycles prevent obtaining tractable results and a transparent understanding of the mechanism of rationing policies. In addition, it will be more difficult to apply dynamic rationing policies with multiple outstanding orders in practice. Since one outstanding order is in fact quite common in reality, this study focuses on this case. As a result, one order cycle can be divided into two parts by the time of releasing an order, namely *period before order release* and *period of lead time*, indicated in Fig. 1.

An order cycle can be viewed as a regenerative process. In this paper, we set reorder point s as the regeneration point. According to this setting, the expected total cost in a cycle consists of two parts, namely the cost in the period before order release and the cost during lead time. Such a structure allows us to calculate the cost in two steps. Melchiors (2003) sets inventory level Q as the regeneration point s, which leads to three steps to calculate the total cost. The cost during the period before order release is divided into two parts, associated with Q to $s+Q$ and $s+1$ to Q. Such a structure was suggested by Melchiors (2003) to facilitate the modelling of policy iteration method. We in fact find that it is better to model the cost in two parts. Its relative simple structure also helps us analyse the problem.

Inventory rationing is employed to improve the system performance in both periods of one order cycle. Once a demand occurs, a decision is made on whether to satisfy it or to reject it. Our objective is to optimise the ordering policy together with the rationing policy in order to minimise the expected average cost. Apart from the penalty cost π_j, ordering cost K and holding cost h are also included. We then introduce the rationing decisions and formulate the related costs and period length in two periods.

2.1. Period before order release

This period starts right after replenishment arrives with inventory level $Q+w$, where w is the leftover inventory at the end of lead time. The period ends with order release, when the inventory level reaches the reorder point s. In this period, the inventory level drops unit by unit. As the demand is Poisson distributed, the length of this period is stochastic.

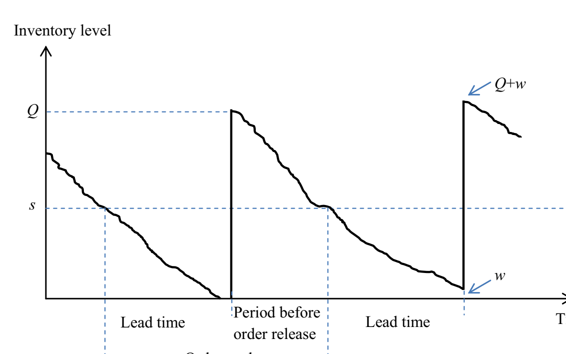

Fig. 1. System dynamics and two periods.