



# Cost minimizing target setting heuristics for making inefficient decision-making units efficient

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## ABSTRACT

A problem of increasing the efficiencies of inefficient decision-making units (DMUs) with known marginal costs of reducing inputs is considered. It is shown that the problem is difficult to solve exactly, and has exponential computational complexity. Two heuristics (Heuristic-I and Heuristic-II) that consider cost minimizing DEA frontier projections for inefficient DMUs are proposed. The proposed heuristics are tested by using several simulated and real-world datasets with linear and Fibonacci marginal input reduction costs. The results of the proposed heuristics are compared with traditional cost insensitive DEA frontier projections, and it is shown that Heuristic-I results in approximately 18–58% lower costs.

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## 1. Introduction

Data envelopment analysis (DEA) models can provide either input or output targets for inefficient decision-making units (DMUs) so that they can be efficient if they achieve the proposed input or output targets (Beasley, 2003). If the marginal costs of reducing inputs or increasing outputs are known, then overall cost minimizing targets can be identified on the efficient DEA frontier. Care must be exercised to identify these targets on the DEA frontier so that the monotonicity assumption of the production process is not violated. Beasley (2003) argues that target setting research has received limited attention and summarizes only a few studies that have addressed this issue.

In this paper, we propose two different cost minimizing target setting heuristics to make all inefficient DMUs efficient while preserving the monotonicity assumption of the underlying production process. Unlike traditional DEA targets that are obtained by identifying a reference set of efficient DMUs, our cost minimizing target setting heuristic investigates a section of the DEA frontier that preserves the monotonicity assumption and may identify targets by using DMUs that are not in the efficient reference set DMUs. We illustrate that searching this section of the DEA frontier for low cost targets for each inefficient DMU is computationally expensive with exponential computational complexity; thus, the procedures that we propose are heuristic procedures. To illustrate that the proposed procedures are more useful than traditional DEA target projections

using only the reference set, we benchmark the overall projection cost by using the procedures against the overall projection cost of traditional DEA target projections by using reference set DMUs. Traditional DEA projections using reference set DMUs do not consider the marginal costs of reducing inputs or increasing outputs and thus, technically, these projections are cost-blind. However, any new procedure should be able to outperform the overall cost of achieving these cost-blind projection targets to be effective. As noted in the previous paragraph, few studies have attempted to address a similar problem, and none of these studies has attempted to preserve the monotonicity property of the production process to obtain these targets. In our paper, we preserve this property of the production process while setting targets that minimize the overall organizational cost of improving the efficiency of inefficient DMUs. Additionally, our procedures guarantee that the cost minimizing targets will make all DMUs in the dataset efficient with respect to original efficient set of DMUs.

The primary motivation for this research is to allow the use of DEA as an organizational tool for cost/benefit analysis. For example, Sherman and Gold (1985) argue that in the banking industry, a 1% decrease in operating expenses resulted in more than a 2% increase in net income and earnings per share for Citicorp. Therefore, while it may be clear that reducing inputs is beneficial for an organization, reducing inputs still incurs additional administrative, training and managerial costs, and ideally, an organization may prefer to reduce these costs as well to maximize profitability. The problem becomes significant when multiple inputs and multiple outputs are considered where unequal marginal costs may be associated with reducing inputs or increasing outputs.

At the outset, we want to clarify that our paper is different from a related popular theme in the DEA literature on fixed cost allocation

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(Cook and Kress, 1999; Beasley, 2003; Jahanshahloo et al., 2004; Lin, 2011; Hosseinzadeh Lotfi et al., 2013). Unlike the fixed cost allocation literature, efficient DMUs in our study are never allocated any new resource inputs and only inefficient DMUs receive additional resource attention to improve their efficiencies. Furthermore, any resources allocated to improve the efficiency of a DMU are not considered to be additional inputs for these DMUs in part because these additional resources are never applicable for efficient DMUs in the dataset. In other words, efficient DMUs receive no resource allocation, while highly inefficient DMUs may receive a high resource allocation to improve their efficiency. In either case, the objective of our research is to minimize the overall resource allocation cost for making all DMUs efficient. The concept of efficiency invariance applicable in fixed allocation research is inapplicable in the current research because our objective is to make all DMUs efficient with reference to original efficient set of DMUs.

We assume that the marginal costs of reducing inputs or increasing outputs are known and available. Golany et al. (1993) make a similar assumption. However, if these costs are not available, then we assume that managers show a predilection towards some input/output variables that may be assigned lower costs, with higher costs assigned to the rest of the variables (Yan et al., 2002).

The rest of the paper is organized as follows. In Section 2, we illustrate that the computational complexity of solving the cost minimizing target setting problem is exponential; next, we describe our heuristics and illustrate their application by using a simple example. In Section 3, we describe a procedure to generate simulated datasets for extensive experimentation and describe the results of our experiments on simulated and real-world datasets. In Section 4, we conclude our paper with a summary and provide directions for future research.

## 2. Cost minimizing target setting heuristics for making inefficient DMUs efficient

DEA was introduced by Charnes et al. (1978). In its ratio form, the DEA model has multiple solutions. To reduce these multiple solutions to a unique representative solution and to convert the ratio form of DEA model into a linear fractional programming problem, a constraint related to either weighted set of outputs or weighted set of inputs is set equal to one. Thus, all DEA models can be classified as either input oriented or output oriented. For an inefficient DMU, an input oriented DEA model identifies target values of inputs to make the DMU fully efficient. Similarly, the output oriented model identifies target output values to make an inefficient DMU efficient.

To describe our heuristic, we assume an input-oriented variable returns to scale DEA model (Banker et al., 1984). Assume  $n$  DMUs with each  $DMU_j$ , where  $j \in \{1, \dots, n\}$ . Each uses  $m$  inputs  $x_{ij}$  ( $i = 1, \dots, m$ ) to produce  $s$  outputs  $y_{rj}$  ( $r = 1, \dots, s$ ); thus, the relative efficiency of  $DMU_o$ , where  $j = o \in \{1, \dots, n\}$ , represented as  $\theta_o$ , can be computed by solving the following linear program (LP):

$$\text{Minimize } \theta_o, \quad (2.1)$$

Subject to:

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta_o x_{io}, \quad i = 1, \dots, m, \quad (2.2)$$

$$\sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s, \quad (2.3)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (2.4)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n. \quad (2.5)$$

Constraints (2.2) and (2.3) ensure that input and output space for all observations is convex and that optimally efficient DMUs are taken from the boundary of the convex hull formed on the input and output space defined by all DMUs. For an inefficient  $DMU_o$  with  $\theta_o^* < 1$ , its projection target on the efficiency frontier is given by a point  $[\sum_{j=1}^n \lambda_j^* x_{1j}, \dots, \sum_{j=1}^n \lambda_j^* x_{mj}]^T$ . The reductions in the inputs of  $DMU_o$  necessary to make it efficient would be the values of the individual components of vector  $[(x_{1o} - \sum_{j=1}^n \lambda_j^* x_{1j}), \dots, (x_{mo} - \sum_{j=1}^n \lambda_j^* x_{mj})]^T$ .

Assuming  $\delta_{io} = (x_{io} - \sum_{j=1}^n \lambda_j^* x_{ij})$  and a marginal input cost reduction vector  $c = [c_1, \dots, c_m]^T$ , the total cost of making an inefficient  $DMU_o$  efficient would be  $\sum_{i=1}^m c_i \delta_{io}$ . Assuming that a set  $E$  consists of all the DMUs in the dataset that are efficient with their  $\theta^* = 1$ , then the total cost to make all inefficient DMUs in the dataset efficient is given by  $\sum_{j=1, j \notin E}^n \sum_{i=1}^m c_i \delta_{ij}$ . We call this overall cost the cost-insensitive or cost-blind correction cost incurred by the organization to make all inefficient DMUs efficient. We seek a heuristic that lowers the values of  $\delta_{ij}$  for  $j \notin E$  so that, if possible, the overall cost can be lowered below the cost obtained by the cost-blind DEA target projection cost, while ensuring that the monotonicity property of the production process is not violated and that the inefficient unit becomes efficient once it achieves the new proposed target input values.

When cost minimizing projections are considered for an inefficient DMU, its original cost-blind reference set of efficient DMUs may or may not be useful in determining the final lowest-cost blind projections. For an inefficient DMU, an exact search procedure must consider multiple LP and DEA formulations (described later in this section) for each different combination of DMUs in the efficient set  $E$ . If  $|E|$  denotes the total number of efficient DMUs then we can create a binary vector  $\xi_s \in \{0,1\}$ , where  $s = \{1, \dots, |E|\}$ , to indicate whether a particular DMU “s” is included in the LP/DEA formulation to compute cost minimizing projections. When  $\xi_s = 1$  the DMU included in computing cost minimizing projection, otherwise  $\xi_s = 0$ . There are a total of  $2^{|E|}$  such combinations. When we eliminate one combination where all  $\xi_s = 0$  and  $|E|$  combinations where exactly one  $\xi_s = 1$  with  $\sum_{s=1}^{|E|} \xi_s = 1$ , we have to consider exactly  $2^{|E|} - 1 - |E|$  combinations of LPs and DEAs that have to be solved per inefficient DMU to solve the problem exactly. Thus, the overall computational complexity to solve the problem exactly is  $O(2^{|E|})$  per inefficient DMU, and is exponential. For datasets where  $|E| = 30$ , over one billion LP and DEA formulations have to be solved for each inefficient DMU. We ignore any further discussion on how all these combinations can be generated, and direct the interested reader to the Pendharkar (2013) study where a related depth first search (DFS) procedure is described to generate DEA ensembles combinations exhaustively. Such DFS procedure can be easily used to generate exhaustive combinations to exactly solve cost minimizing target setting problem. In the heuristics that we propose in this section, we only consider a subset of these combinations where  $\sum_{s=1}^{|E|} \xi_s = 2$ , i.e., we consider combinations of efficient DMUs in pairs. The computational complexity of our heuristic procedures is  $\binom{|E|}{2} = O(|E|^2)$  per inefficient

DMU, and is polynomial. Assuming a dataset with 500 DMUs containing 30 efficient DMUs and 470 inefficient DMUs, an exhaustive search will require solving over 470 billion LP and DEA models, whereas our polynomial heuristic procedure will require solving 423,000 LPs and DEA models. Large scale DEA problems containing a few hundred DMUs to a few thousand DMUs are typically found in component based software

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