Int. J. Production Economics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Algorithms for asset replacement under limited technological forecast

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ARTICLE INFO

Article history:
Received 21 January 2014
Accepted 27 August 2014

Keywords:
Asset replacement models
Economic life
Technological change
Finite horizon
Optimization

ABSTRACT

The optimal asset replacement is analyzed when the future course of technological change is known on a limited future horizon. Comparison of factual and desired properties of known replacement methods leads us to the idea of how to improve their efficiency under changing technology reflected in decreasing operating and new asset costs. We introduce new modifications of the economic life and two-cycle variable-horizon methods by correcting their capital recovery factor. Next, we demonstrate that the modified methods deliver solutions equal or close to the infinite-horizon replacement under technological change.

1. Introduction

The optimal replacement of productive assets under changing operating and maintenance costs caused by technological advances is a problem of enormous theoretical complexity and practical importance (Hartman and Tan, 2014). This paper focuses on a theoretic analysis of deterministic asset replacement methods when the future course of technological change is known on a limited forecast horizon only. On the basis of a comparative theoretic analysis and numeric simulation of existing replacement methods, we introduce their new modifications that deliver solutions equal (or close) to the infinite-horizon replacement policy under improving technology. The known replacement algorithms can be grouped in the following categories:

(IH) The *infinite-horizon* minimization of the present value of the total replacement cost chooses the infinite sequence of the future lifetimes of a sequentially replaced asset (Elton and Gruber, 1976; Sethi and Chand, 1979; Bethuyne, 1998; Regnier et al., 2004; Yatsenko and Hritonenko, 2005, 2008, 2010).

(EL) The *economic Life* method minimizes the asset's total equivalent annual cost by choosing the optimal replacement time of current asset (Thuesen and Fabrycky, 1993; Newman et al., 2004; Hartman, 2007). This method is equivalent to the infinite-horizon optimization in the case of stationary asset costs. In contrast to common opinion (Hartman, 2007) that the EL method does not consider improving technology, we demonstrate that it can take the changing cost of new assets into account.

(VH) The *variable-horizon* replacement method of Christer and Scarf (1994) minimizes the annual replacement cost over two future replacement cycles.

(FH) The *fixed-horizon* replacement determines a finite sequence of the future asset replacements to minimize the present value of the replacement cost over a given finite horizon (Hritonenko and Yatsenko, 1996, Scarf and Hashem, 2002). We will skip this method from consideration because it involves significant end-of-horizon effects and is less efficient than the variable-horizon method (Scarf and Hashem, 2002, Yatsenko and Hritonenko, 2005, 2011).

The goal of this paper is to construct replacement methods that use limited technological forecast data but produce the best results (at least, the time of first replacement) for the infinite-horizon technological forecast. Correspondingly, our ideal benchmark problem is the infinite-horizon replacement. It is known (Regnier et al., 2004, Yatsenko and Hritonenko, 2011) that the economic life and variable-horizon methods under technological change produce results different from the infinite-horizon replacement. At the same time, comparing factual and desired properties of these methods allows us to improve their efficiency under continuing technological improvement. Specifically, we introduce modified economic life and variable-horizon methods with a corrected capital recovery factor that compensates the influence of further unconsidered horizon. We prove that both modified methods deliver the best (infinite-horizon) solution if the observed technological change is exponential and affects equally both the operating and new asset costs (the so-called proportional technological change). Namely, the suggested methods find the

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http://dx.doi.org/10.1016/j.ijpe.2014.08.020
0925-5273/

infinite-horizon optimal asset lifetime for proportional technological change and arbitrary age-dependent profile of deterioration.

In the case of non-proportional exponential technological change, the optimal asset lifetime is non-constant and depends on the cost components: the optimal lifetime of sequentially replaced assets increases if the operating cost decreases faster than the new asset cost, and converse (Regnier et al., 2004, Yatsenko and Hritonenko, 2011). Then, we use numeric simulation to show that the modified VH method is the best option and produces the solution closest to the infinite-horizon optimization.

The paper is as follows. Section 2 describes the economic life, variable-horizon, and infinite-horizon replacement methods and introduces their modifications in a continuous-time serial replacement model. Section 3 provides necessary background for theoretical analysis. Section 4 compares the methods analytically and numerically and demonstrates a superior performance of the modified replacement methods under various scenarios of improving technology. Section 5 concludes and gives some practical recommendations.

2. Formulation of asset replacement methods under study

Let us consider a firm that needs to replace periodically a single asset with new assets that perform identical operations but have different replacement costs. We will describe this process in the continuous time $t \in [0,\infty)$. The changing economic-technological environment is represented by the following functions:

- $P(t)$ is the cost (purchase price and installation cost) of a new asset at time t (of vintage t).
- $A(t,u)$ is the operating and maintenance cost at time u for the asset bought at time t. Then, the variable $u-t$ is the age of the asset, $0 \le u-t \le M$, and M is the maximal physical service life of assets.
- $S(t,u)$ is the salvage value of the asset of age $u-t$ bought at time t, $0 \le S(t,u) < P(t)$.

The technological change leads to the availability of newer assets that require less maintenance and/or are less expensive, so $P(t)$ and $A(t,u)$ decrease in t. The maintenance cost $A(t,u)$ usually increases in the age $u-t$ (as the asset becomes older) because of physical deterioration, however, it can also decrease because of learning (Goetz et al., 2008). The general form of the function $A(t,u)$ can depict various hypotheses of deterioration and learning.

To calculate actual replacement costs over a finite horizon, the replacement theory commonly uses the capital recovery factor $R(r,T)$ that converts the present value of a certain cost over a specified future interval into the sequence of equivalent annual costs. Under the assumption of continuous compounding, the annual capital recovery factor over the interval $[0,T]$ is as follows:

$$R(r,T) = \frac{r}{1-e^{-rT}} \tag{1}$$

where $r>0$ is the instantaneous discount rate. One can see that $R(r,T) \to 1/T$ at $r \to 0$, which is consistent with the continuous-time analysis of the zero-discounting case by Scarf and Hashem (2002).

To describe the process of sequential replacement of the single asset with a new asset, we introduce the endogenous lifetime L_k of the k-th asset, $k=1,2,\ldots$. For clarity, we assume that the first asset is purchased at time $t=0$ and will be replaced at the end of its lifecycle, then the time of introducing the first asset is $\tau_0=0$ and the time of its replacement with the second asset is $\tau_1=L_1$. Correspondingly, the time τ_k of the replacement of k-th asset with $(k+1)$-th asset is as follows:

$$\tau_k = \tau_{k-1} + L_k = \Sigma_{j=1}^{k} L_j. \tag{2}$$

The asset replacement cost: The present value of the total replacement cost of the k-th asset, $k=1,2,\ldots$, over its future lifetime L_k is calculated at a given industry-wide discount rate $r>0$ as follows (Regnier et al., 2004, Hartman, 2007):

$$PV_k(L_k,\tau_{k-1}) = e^{-r(\tau_{k-1}+L_k)}[P(\tau_{k-1}+L_k) - S(\tau_{k-1},\tau_{k-1}+L_k)] + \int_{\tau_{k-1}}^{\tau_{k-1}+L_k} e^{-ru} A(\tau_{k-1},u)\,du \tag{3}$$

The first term of (3) represents the discounted cost of the new asset minus the discounted salvage value of the current asset, and the integral is the discounted maintenance costs over the future lifetime of the current asset. Next, we provide mathematical formulations of the replacement methods under study.

2.1. Infinite-horizon replacement

The infinite-horizon replacement method (Regnier et al., 2004; Hartman, 2007; Hritonenko and Yatsenko, 2007, 2008) assumes the knowledge of external technological parameters P, A, and S on the infinite horizon $[0,\infty)$ and determines the infinite optimal sequence of consecutive asset lifetimes L_k, $k=1,2,\ldots$, that minimizes the present value of the total replacement cost over $[0,\infty)$:

$$\mathrm{PV}_\infty(L_1{}^*, L_2{}^*, \ldots) = \min_{L_k, k=1,\ldots; 0<L_k\le M} \mathrm{PV}_\infty(L_1, L_2, \ldots), \tag{4}$$

$$\mathrm{PV}_\infty(L_1, L_2, \ldots) = \sum_{k=1}^{\infty} \mathrm{PV}_k(L_k, \tau_{k-1}), \tag{5}$$

where PV_k is given by (3) and τ_k is determined from (2).

In contrast to the infinite-horizon optimization, other replacement methods work in the case of a limited technological forecast. Namely, we assume that the technological parameters $P(t)$, $A(t,u)$, and $S(t,u)$ are known for $0 \le t \le u \le T < \infty$ on some finite future interval $[0,T]$, where the value T is not smaller than the unknown lifetime L_1 of the current asset. To ensure that, it is enough to assume $T \ge M$.

2.2. Economic life replacement

The economic life (EL) method determines the first asset lifetime that minimizes the equivalent annual cost of the first asset replacement (Thuesen and Fabrycky, 1993).

$$C_1(L_1) = R(r,L_1)\mathrm{PV}_1(L_1,0). \tag{6}$$

By the EL method, the optimal lifetime EL_1 of the first asset is determined as follows:

$$\mathrm{EL}_1 = \arg\min_{0<L\le M} C_1(L). \tag{7}$$

To find the first optimal lifetime EL_1, it is enough to know the cost $P(t)$ and sequences $S(0,t)$ and $A(0,t)$ over the future interval $[1,\mathrm{EL}_1]$ only. In the general case, the EL method produces different optimal lifetimes EL_1, $\mathrm{EL}_2,\ldots$, for sequentially replacements $k=1,2,3,\ldots$ of the asset. Finding the first optimal lifetime EL_1 is the most relevant task in engineering practice.

A common consensus in the operations research replacement theory is that the EL method cannot take technological change into account. Fortunately, it is true only partially. Indeed, the above version (6) of the EL method assumes replacement at the end of the current asset lifecycle and so, in fact, considers possible technological improvement as the change of the new asset cost $P(L_1)$. At the same time, the EL method (4) and (5) does not consider improvements in the maintenance cost at all. In Section 2.4 below, we will offer a modified EL method that addresses this drawback.