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A two-echelon production-inventory model for deteriorating items with multiple buyers

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ABSTRACT

In a production-inventory system, the manufacturer produces the items at a rate, e.g. *R*, dispatches the order quantities to the customers in specific intervals and stores the excess inventory for subsequent deliveries. Therefore each inventory cycle of the manufacturer can be divided into two phases, first is the period of production, the second is when the manufacturer does not do any production and utilises the inventory that is in stock. One of the challenges in these models is how to obtain the inventory level of the supplier when there is deterioration. The existing literature that considers multi-echelon systems (including models with single-buyer or multi-buyer), analyses the deterioration/inventory cost of these echelons with the assumption of having huge surplus in production capacity. Then it seems acceptable to drop part of the production period which is for producing the first batch(s) for buyer(s) at the beginning of each production period. In this paper we develop a single-manufacturer, multi-buyer model for a deteriorating item with finite production rate. We also relax the assumption on the production capacity and find the average inventory of the supplier. It is shown that in case the production rate is not high, the existing models may not be sufficiently accurate. It is also illustrated that these models are more applicable to inventory systems (and not production-inventory) as they result in fairly accurate solutions when the manufacturer has much higher production capacity compared to the demand rate. Also a sensitivity analysis is conducted to show how the model reacts to changes in parameters.

1. Introduction

Inventory management literature is largely based on the assumption that an item in stock suffers no loss in quality. This, however, cannot hold in practice for many products. In order to address this fact, a subset of the literature studies *deteriorating items*. The deteriorating item inventory models have increasingly drawn attention in recent years. The main stream of this literature considers that a percentage of on-hand inventory goes bad.

This literature mainly includes single-echelon inventory models. Recent research papers on deteriorating item inventory strive to go beyond a single echelon and address a supply chain model in order to make it more applicable to real-world problems. The number of studies on multi-echelon supply chain however is few.

One of the challenges in modelling a multi-echelon supply chain of a deteriorating item is to evaluate the inventory level at the supplier, specially when there are multiple buyers that can have multiple replenishments. Ghiami et al. (2013) obtain the exact *inventory level* of the supplier of a two-echelon distribution system (a single-buyer, single-supplier model). In this model however the production rate is infinite which means that the supplier receives the batches in lots. Also having multiple buyers adds to the complexity of the inventory level at the supplier.

Single-manufacturer multi-buyer models are few in the literature. Assuming finite production rate (production-inventory model) with multiple buyers makes the inventory level at the supplier complex to find. In order to analyse this complexity, the few research papers on production-inventory multi-echelon supply chain models for deteriorating items use approximations which give sufficiently accurate results under the assumption of huge surplus in production capacity. These assumptions however make the model less practical for some cases.

In this paper a multi-echelon supply chain with finite production rate is developed and analysed. Moreover it is shown that the existing models can find fairly accurate solution under specific assumptions. By using a numerical example, it is also illustrated that if those circumstances do not hold, the existing models fail to

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find accurate solutions whereas the current model can do so. The significance of these errors is also depicted.

2. Literature review

One of the first studies which analyse a multi-echelon supply chain for a deteriorating item with finite production rate is done by Yang and Wee (2000). For an overview of the deteriorating item's literature see Nahmias (1982), Raafat (1991), Goyal and Giri (2001), Li et al. (2010), and Bakker et al. (2012). Yang and Wee (2000) consider single-buyer single-vendor where the vendor produces the items with a finite production rate. The objective is to minimise the total cost function of the supply chain. Later Yang and Wee (2002) extend the work done by Yang and Wee (2000) by considering multiple buyers and aim to minimise the total cost of the system. In their model, Yang and Wee (2002) assume that there is a huge surplus in production capacity therefore they neglect the production time that is needed to produce the items for the buyers before the next production-inventory cycle starts. Rau et al. (2003) develop a single-supplier, single-manufacturer, single buyer model. Similar to Yang and Wee (2002), Rau et al. (2003) implicitly assume that the production rate is significantly larger than the demand, therefore they drop part of the manufacturer's production period. Law and Wee (2006) consider a single-buyer, single-vendor supply chain which produces and delivers a product of which the raw material is livestock. The manufacturer buys young livestock and grows them, the mature livestock is then used to make food. This food is delivered to the buyer in batches. In their research, Law and Wee (2006) consider the time value of money by discounting the cost with a specific rate, ultimately minimising the total cost of the system. Taking a discounted cash flow approach, Lo et al. (2007) model a production-inventory system which consists of one buyer and one manufacturer. Similar to the previous research works, Lo et al. (2007) aim to minimise the total cost of the system.

Yan et al. (2011) examine an integrated single-buyer, single-supplier model with finite production rate using an approximated method in order to simplify the problem. In order to keep the error of this approximation small, Yan et al. (2011) assume the deterioration rate to be very small. Yu et al. (2012) study a supply chain in which the buyers are assumed to have the same inventory period and the supplier has a huge surplus in production capacity. The model is developed and analysed based on the assumption that the manufacturer has a large production capacity compared to the total demand. Also the production set-up cost is considered to be small so the supplier starts and stops the production process very frequently. These assumptions do not allow any stock accumulation at the supplier as he starts producing the items just some time before batches are delivered. Yu et al. (2013) develop a similar model in which there is only one buyer. The deterioration rate is considered to be based on a Weibull distribution. Yu (2013) develops a single-buyer, single-supplier model and suggests a collaborative strategy for setting the inventory policies. Lee and Kim (2014) analyse a single-vendor, single-buyer supply chain for a deteriorating item considering a probability for items to be defective as a failure in the production process. In their research, Lee and Kim (2014) consider the inventory level of the supplier as in Yang and Wee (2000, 2002).

The literature shows that there are few studies on single-manufacturer multi-buyers models. In this paper a similar model to Yang and Wee (2002) and Yu et al. (2012) is developed. In order to make a more generic and practical model, however we assume that the buyers can have different inventory periods and also the assumption of having a huge surplus in production capacity is relaxed. Thereafter the result of the current model is compared with the existing literature.

3. Model

In this paper a supply chain including one manufacturer and N buyers is considered which delivers an item with constant deteriorating rate of the on-hand inventory. The manufacturer starts the production and stores the items sometime (T_3) before the first batch is sent to all the buyers at $t=0$. He continues the production till $t=T_1$, while storing the excess inventory for next deliveries. At time $t=T_1$, the manufacturer starts a non-production period which lasts till $t=T_1+T_2$ which is followed by a production period with length of T_3 during which the manufacturer produces items for the next inventory cycle. This production-inventory process repeats itself infinitely. Shortages are not allowed neither at the supplier nor the buyers. There is no replacement for the deteriorated items. The production rate is assumed to be greater than the sum of the demand at all the buyers. The notations used in this model are as follow:

θ	the deterioration rate
N	number of the buyers
d_i	the demand rate at buyer i, $i=1,2,\ldots,N$
p	the production rate
T	the production-inventory cycle at the manufacturer, where $T=T_1+T_2+T_3$
T_1	the production period after the first batch sent to all the buyers
T_2	the non-production period during T
T_3	the production period to produce enough inventory to send to all the buyers at the end of T
$\tilde{I}_{v1}(t_1)$	the echelon stock at the supplier between $t_1=0$ and $t_1=T_1$
$\tilde{I}_{v2}(t_2)$	the echelon stock at the supplier between $t_2=0$ and $t_2=T_2$
$\tilde{I}_{v3}(t_3)$	the echelon stock at the supplier between $t_3=0$ and $t_3=T_3$
$\tilde{I}_{bi}(t)$	the echelon stock at retailer i between $t=0$ and $t=T/n_i$
n_i	number of replenishments of buyer i during T
$I_{v1}(t_1)$	the physical inventory at the supplier between $t_1=0$ and $t_1=T_1$
$I_{v2}(t_2)$	the physical inventory at the supplier between $t_2=0$ and $t_2=T_2$
$I_{v3}(t_3)$	the physical inventory at the supplier between $t_3=0$ and $t_3=T_3$
$I_{bi}(t)$	the physical inventory at retailer i between $t=0$ and $t=T/n_i$, $I_{bi}(t)=\tilde{I}_{bi}(t)$
I_{mi}	the maximum inventory level (order quantity) of buyer i
p_v	the unit production cost at the supplier
p_b	the unit purchasing price for the buyers
F_v	the holding cost per unit of currency per time unit at the supplier
F_b	the holding cost per unit of currency per time unit at each buyer
C_{sv}	the set-up production cost for the supplier
C_{sb}	the fixed ordering cost for each buyer
TP	the average profit of the supply chain
TP_v	the average profit of the supplier
TR_v	the average revenue of the supplier
TC_v	the total average cost of the supplier
HC_v	the average holding cost of the supplier
DC_v	the average deterioration cost of the supplier
SC_v	the average ordering cost of the supplier

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