Computers & Geosciences 44 (2012) 136–145

Contents lists available at SciVerse ScienceDirect

Computers & Geosciences

journal homepage: www.elsevier.com/locate/cageo

Simulation of axi-symmetric flow towards wells: A finite-difference approach

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ARTICLE INFO

Article history:
Received 10 February 2011
Received in revised form
4 August 2011
Accepted 5 September 2011
Available online 21 September 2011

Keywords:
Non-uniform axi-symmetric flow
Multi-layer systems
Aquifer tests
Finite-difference model
MATLAB tool
MODFLOW

ABSTRACT

A detailed finite-difference approach is presented for the simulation of transient radial flow in multi-layer systems. The proposed discretization scheme simulates drawdown within the well more accurately than commonly applied schemes. The solution is compared to existing (semi) analytical models for the simulation of slug tests and pumping tests with constant discharge in single- and multi-layer systems. For all cases, it is concluded that the finite-difference model approximates drawdown to acceptable accuracy. The main advantage of finite-difference approaches is the ability to account for the varying saturated thickness in unconfined top layers. Additionally, it is straightforward to include radial variation of hydraulic parameters, which is useful to simulate the effect of a finite-thickness well skin. Aquifer tests with variable pumping rate and/or multiple wells may be simulated by superposition. The finite-difference solution is implemented in MAxSym, a MATLAB tool which is designed specifically to simulate axi-symmetric flow. Alternatively, the presented equations can be solved using a standard finite-difference model. A procedure is outlined to apply the same approach with MODFLOW. The required modifications to the input parameters are much larger for MODFLOW than for MAxSym, but the results are virtually identical. The presented finite-difference solution may be used, for example, as a forward model in parameter estimation algorithms. Since it is applicable to multi-layer systems, its use is not limited to the simulation of traditional pumping and slug tests, but also includes advanced aquifer tests, such as multiple pumping tests or multi-level slug tests.

1. Introduction

Simulation of groundwater flow caused by pumping wells or injection wells is a crucial step in the interpretation of aquifer tests. In general, the interpretation method is a combination of a forward mathematical model to simulate the flow and a curve fitting method, which is either a graphical procedure based on type curves or a regression method using a numerical algorithm. In most cases, flow towards or away from a well exhibits radial symmetry, which reduces the governing flow equation by one dimension and justifies the use of an axi-symmetric model. Although radial flow is generally treated by one partial differential equation, a variety of axi-symmetric models are described in the literature (e.g. Butler, 1998; Kruseman and De Ridder, 1990; Reed, 1980), presenting specific analytical solutions determined by the type of flow, the type of test and the particular aquifer system in which the test is performed. These solutions are often obtained by applying integral transforms (e.g. Neuman and Witherspoon, 1969).

Besides these specific analytical models, more generic models have been developed, which solve the radial flow equation either semi-analytically employing integral transforms (e.g. Hemker and Maas, 1987; Veling and Maas, 2009) or numerically applying the finite-element method (e.g. Pandit and Aoun, 1994; Reilly, 1984), the finite-difference method (e.g. Bohling and Butler, 2001; Johnson et al., 2001), or a combination of these methods (e.g. Lebbe, 1999). Generic models offer the flexibility to simulate different types of tests in more complex aquifers considering more realistic boundary conditions using a single code. The model developed by Lebbe (1999), for instance, was applied to interpret multiple pumping tests (Lebbe et al., 1992), combine interpretation of pumping and tracer tests (Vandenbohede and Lebbe, 2003), analyze the upconing of saltwater into extraction wells (Van Meir and Lebbe, 2005), identify microbial aerobic respiration and denitrification kinetics using push–pull tests (Vandenbohede et al., 2008), analyze heat transport during push–pull tests (Vandenbohede et al., 2009), and interpret step-drawdown tests in layered aquifers (Louwyck et al., 2010).

The objective of this paper is threefold. First, a detailed finite-difference formulation is presented for transient axi-symmetric

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doi:10.1016/j.cageo.2011.09.004

groundwater flow in multi-layer systems. The formulation is implemented in the MATLAB tool MAxSym (Louwyck, 2011), which is made available for public distribution from the journal's website. The main advantage of a finite-difference solution is the ability to simulate the variation of the saturated thickness in unconfined top layers. It is also straightforward to consider more realistic boundary conditions at the well and to include the effect of a finite-thickness well skin. Radial distance and time are discretized in logarithmic space to minimize computational efforts and yet obtain accurate results at any distance and any time. The proposed discretization scheme is an improvement over commonly applied finite-difference schemes for the simulation of drawdown within a well, as is important for slug tests.

Axi-symmetric flow may also be simulated using a finite-difference model with rectilinear grid geometry (e.g. Anderson and Woessner, 1992; Halford and Yobbi, 2006; Langevin, 2008; Reilly and Harbaugh, 1993; Samani et al., 2004). The second objective of this paper is to describe a procedure to use MODFLOW 2005 (Harbaugh, 2005) for solving the presented equations, and to apply it for verification of the MATLAB code.

The third objective is to compare the finite-difference solution to existing analytical solutions for homogeneous aquifers: the models of Theis (1935) and Hantush–Weeks (Hantush 1964; Weeks, 1969) for pumping tests and the model of Cooper et al. (1967) and the KGS model (Hyder et al., 1994) for slug tests. Simulation of pumping and slug tests in a multi-layer system is compared to the TTim solver (Bakker, 2010), a semi-analytical solution to the system of differential equations described in Hemker and Maas (1987). The purpose of these comparative tests is to determine whether the presented finite-difference solution is sufficiently accurate to be incorporated as a forward model in parameter estimation algorithms.

2. Statement of problem

Pumping tests and slug tests are two types of aquifer tests that have proven to be effective ways of obtaining reliable values for the hydraulic characteristics of an aquifer system. During a conventional pumping test, water is pumped from a well with a constant discharge, and drawdown is measured in the well and in observation wells at known distances from the pumping well. In a conventional slug test, a small volume of water is suddenly removed from or poured into a well, after which the water level in the well is measured until it returns to the original level. The presented problem description is restricted to over-damped slug tests, which means that inertial forces in the well are not considered.

If no angular variations in hydraulic properties occur, groundwater flow towards or away from a well is axi-symmetric and treated by the following partial differential equation:

$$\frac{\partial}{\partial r}\left(K^r_{(z,r)}\frac{\partial s_{(z,r,t)}}{\partial r}\right)+\frac{K^r_{(z,r)}}{r}\frac{\partial s_{(z,r,t)}}{\partial r}+\frac{\partial}{\partial z}\left(K^z_{(z,r)}\frac{\partial s_{(z,r,t)}}{\partial z}\right)=S^s_{(z,r)}\frac{\partial s_{(z,r,t)}}{\partial t} \quad (1)$$

where r is the radial distance (m), z is the vertical distance (m), t is time (d), s is the drawdown (m), K^r is the radial component of hydraulic conductivity (m/d), K^z is the vertical component of hydraulic conductivity (m/d), and S^s is the specific elastic storage coefficient (m^{-1}). Drawdown $s(z,r,t)$ is defined as the difference between the head $h(z,r,t)$ and the steady initial head $h_0(z,r)$ before the test starts at $t=0$

$$s_{(z,r,t)}=h_{(z,r,t)}-h_{0(z,r)} \quad t\geq 0 \quad (2)$$

Drawdown at the face of the well screen is assumed equal to the change in head $H(t)$ within the well

$$s_{(z,r_w,t)}=H_{(t)} \quad t\geq 0,\quad z_1\leq z\leq z_2 \quad (3)$$

where r_w is the radius of the well screen, which is situated between levels z_1 and z_2.

The initial conditions hold for time $t=0$ when the test starts

$$s_{(z,r,0)}=0 \quad r>r_w \quad (4)$$

$$s_{(z,r_w,0)}=\begin{cases}H_0 & z_1\leq z\leq z_2\\ 0 & \text{elsewhere}\end{cases} \quad (5)$$

Condition (4) states that there is no drawdown in the aquifer when the test starts. For condition (5), a distinction is made between pumping and slug tests. If a pumping test is simulated, initial drawdown H_0 within the well is zero. In case of a slug test, the instantaneous injection or removal of a known volume of water in the well at $t=0$ causes an initial change in head H_0 that differs from zero.

The following boundary conditions hold for $t>0$:

$$2\pi r_w\int_{z_1}^{z_2}K^r_{(z,r_w)}\frac{\partial s_{(z,r_w,t)}}{\partial r}dz=-Q-\pi r_w^2\frac{\partial s_{(z,r_w,t)}}{\partial t} \quad z_1\leq z\leq z_2 \quad (6)$$

$$\frac{\partial s_{(z,r_w,t)}}{\partial r}=0 \quad z<z_1 \text{ or } z>z_2 \quad (7)$$

$$s_{(z,\infty,t)}=0 \quad (8)$$

$$\frac{\partial s_{(0,r,t)}}{\partial z}=\frac{\partial s_{(D,r,t)}}{\partial z}=0 \quad r_w<r<\infty \quad (9)$$

Condition (6) means that the rate of flow of water through the well screen is equal to the sum of the pumping rate Q (m^3/d) and the rate of decrease or increase in volume of water within the well. Elsewhere, the well face is impermeable which is expressed by condition (7). For slug tests, the pumping rate Q is zero, and in case of an injection rate Q is negative. From (8) one derives that the aquifer has an infinite extent with drawdown at infinity equal to zero. Condition (9) states that the aquifer is confined and has thickness D, where the lower aquifer boundary is at level $z=0$ and the upper boundary at level $z=D$. When $z_1=0$ and $z_2=D$, the well is fully penetrating.

3. Finite-difference approximation

An accurate finite-difference solution to the problem stated above is implemented in the object-oriented MATLAB tool MAxSym (Louwyck, 2011), which can be downloaded from the journal's website. This section gives a detailed description of MAxSym's finite-difference formulation.

3.1. Discretization

The vertical distance z is discretized by subdividing the aquifer into n_l layers with constant thickness, where $D(i)$ is the thickness of layer i. The nodes are positioned at the center of each layer. Layers are numbered from the top of the aquifer to the bottom. Radial distance r and time t are discretized in logarithmic space by applying the following definitions:

$$r_{(j)}=r_{(j-1)}a_r \quad j=2,3,\ldots,n_r \quad (10)$$

$$t_{(k)}=t_{(k-1)}a_t \quad k=2,3,\ldots,n_t \quad (11)$$

Eq. (10) defines a series of n_r nodal circles with radii $r(j)$, where a_r is a constant greater than 1 and smaller than 2. Each nodal circle j lies inside ring j at a radial distance equal to the geometric mean of inner radius $r(j-0.5)=r(j)a_r^{-0.5}$ and outer radius $r(j+0.5)=r(j)a_r^{0.5}$ (Fig. 1). The first nodal circle is inside the well and its radius is computed as $r(1)=r_wa_r^{-0.5}$. Hence, the outer radius of the first ring is equal to the well radius r_w. Similarly, Eq. (11) defines a series of n_t time steps $t(k)$ where a_t is a constant greater than