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Optimizing mining rates under financial uncertainty in global mining complexes

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ABSTRACT

This paper presents a distributed and dynamic programming framework to the mining production rate target tracking of multiple metal mines under financial uncertainty. A single mine's target tracking is stated as a stochastic optimization problem and the solution is obtained by solving the dynamic program which gives the optimal production rate schedule of each mine as a Markovian feedback control on the price process. The global solution is distributed on multiple mines by a policy iteration method, and this iterative method is shown to provide the unique equilibrium among Markovian strategies. Numerical results confirm the efficacy of the proposed global method when compared to individual optimization of mining rate target tracking.

1. Introduction

A mining complex is composed of multiple mines, material types, and several processing streams including stockpiles. A global optimization framework for a mining complex should take into account the dynamics and mutual constraints of the overall complex. In this paper, we investigate mining production rate target tracking of multiple metal mines in a mining complex over the life of operations from which life of mine schedules are generated. It is important to maintain a steady mining rate during the life of mine since moving mining equipment and relocating personnel is costly. However, in changing market dynamics, the trade-off between following the planned mining rate and cost of rate change forms a dynamic stochastic optimization problem, which is termed mining rate tracking problem.

Three fundamental properties affect mining rate planning in a mining complex: metal uncertainty, financial uncertainty and inter-dependence of mines in a mining complex. First, since the metal content of each mining block is not known, the associated financial value of a block is stochastic. Traditionally, to overcome this stochasticity, scenario methodologies are applied (Ramazan and Dimitrakopoulos, 2013; Boland et al., 2008; Meagher et al., 2009), the cost function is assumed to be linear and stochastic mixed integer programming (SIP) based solutions are adopted. Since the number of mining blocks is usually very large, heuristic approaches have been applied (Lamghari and Dimitrakopoulos, 2012). An alternative is to develop sequential models to form a complete plan, as per Lerchs and Grossmann (1965) and Whittle (1988). We take a similar approach here and extend this sequential approach to multiple mines in a single mine complex with the novelty that (i) it is dynamic programming based and (ii) it takes global dependences into account in an iterative manner.

Secondly, the price of the metal is a stochastic process. Since mining rate tracking is a horizon optimization problem and the price is observed progressively on the horizon, this introduces feedback controls to the tracking problem. Lastly, there are mutual constraints that have to be addressed by all mines such as stockpiles and processing destinations that are common parts of a mining complex. Therefore, a global optimization framework is needed.

Even though financial uncertainty has been addressed less than geological uncertainty in the mining literature (Godoy, 2003), there has been progress in the recent years. Simulation-based approaches have been presented by Abdel Sabour and Dimitrakopoulos (2011), and a methodology to quantify the effect of price uncertainty within reserve estimates has been given by Evatt et al. (2012). A graph-based parametric maximum flow algorithm for developing ultimate pit limit and phase design under metal and financial uncertainty has been presented by Asad and Dimitrakopoulos (2013).

The problem discussed herein may be seen as a sub-problem under the larger problem named production scheduling of a mining complex under financial uncertainty. Ideally, this problem should be solved globally in a single stochastic mixed integer

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program. However, the computational complexity is insurmountable. In the work presented here, a distributed approach rather than a centralized framework is taken. The problem is divided into four phases, the necessary and salient dependences between the phases are established, and then solved iteratively. Phase 1 is the mining pit limit calculation of each mine and phase 2 is the calculation of individual mining production rate target functions which includes identifying independent and dependent constraints. In phase 3, mining rate trackings are solved globally for all mines considering financial uncertainty. Phase 4 includes the calculation of individual production schedules considering metal uncertainty. Note that metal uncertainties are independent of each other and independent of the financial uncertainty, which allows parallel computation in phase 4. It should be stressed that without this iterative approach it would not be possible to employ parallelization in the scheduling phase, and the resulting single stochastic mixed integer program would have an enormous number of variables and constraints, which necessarily would lead to intractability. This paper proposes a solution to the phase 3 described above. The proposed framework provides a significant reduction in computation of decision making in an environment where every decision is dependent on volatile prices. Once the extraction rates are calculated the production scheduling can be done individually for each mine taking into account each mine's individual geological uncertainties, which are in principle independent of each other.

In this paper, an optimal control framework is developed to address financial uncertainty and global optimization in mining production rate tracking for multiple mines in a mining complex. Note that, in previous work, receding control has been applied to mine production scheduling by Goodwin et al. (2006) and Rojas et al. (2007) in a deterministic framework. Herein a stochastic optimization framework is presented where individual target tracking on the horizon is shown to be in the class of Markovian feedback controls, where the price process is progressively measured, but only the instantaneous price is employed to calculate the mining rate. The stochastic properties of the price process are handled in a dynamic program. Since the individual dynamic optimizations of the mines are coupled a distributed policy iteration method is provided, and it is shown that successive iterations converge to a unique fixed point which represents the unique Nash equilibrium.

The assumptions below are made in the present work.

A1: The existence of a target extraction rate function is assumed for each mine parameterized by the price process and the extraction rates of other mines, denoted as $\Psi^k(p_t, \overline{x}_t), t \geq 0$ where $p_t, t \geq 0$ is the price process and $\overline{x}_t, t \geq 0$ denotes the average extraction rate of all mines. The target extraction rate function is strictly increasing in price and strictly decreasing in the average extraction rate of all mines. Several other parameters can be injected into this function such as the overall estimated value of the mining complex, the relative complexity of the transportation for each mine, etc. The key idea here is to group dependences into a single dynamics function where dynamic optimization can be applied. The determination of the structure of the target extraction rate function precisely requires a sensitivity analysis with respect to the selected parameters and is beyond the scope of the paper.

The mines are not independent; for instance, if all the mines increase their extraction rates, even though mines respect their individual constraints, the global constraints could be violated or stockpile capacities that are commonly used could be exceeded. It is to be noted that the results of the paper hold in the case when **A1** is generalized to a more general functional where (a) the parameter set is finite and (b) Markovian property is not violated. **A1** has been established for notational brevity.

A2: It is assumed that the metal price follows a stochastic differential equation (Schwartz, 1997) which is subject to Brownian increments, nowhere differentiable, Markovian and given by

$$dp_t = f^p(p_t;\mu)dt + \sigma(p_t)dw_t, \tag{1}$$

where $f^p(p;\mu)$ is the drift and $\sigma(p)$ is the volatility, whereas w is a standard Wiener process (Brownian motion).

The time evolution of the probability density function $\zeta(t,p_t)$ of the metal price that is modeled through (1) is given by the Fokker–Planck equation which in physics provides the evolution of the probability density function of the velocity of a particle given by

$$\partial_t \zeta(t,p_t) + \partial_p[f^p(p_t;\mu)\zeta(t,p_t)] - \frac{1}{2}\partial^2_{pp}\sigma(p_t)^2\zeta(t,p_t) = 0, \tag{2}$$

where a closed form solution may exist depending on the properties of f^p and σ. Since the time varying distribution of the metal price is explicitly stated through a partial differential equation (PDE), stochastic mining rate target tracking can be simply formulated as a stochastic mixed integer program with recourse. However, despite its simple model, the solution would be hit by the curse of dimensionality in the uncountable and unbounded state space, and it would be computationally intractable to provide Monte Carlo solutions even if the distribution (2) is very roughly sampled. In this paper, optimal mining rate tracking is solved via a dynamic program formulation solvable in closed form; therefore the approach offers a significant complexity advantage.

Classical optimization and control theory studies problems with a single decision maker and offers tools and algorithms that can guarantee a certain performance and robustness. Decentralization of a global system immediately poses new problems to be solved such as those raised by the well-known Witsenhausen counterexample (Witsenhausen, 1968), or the stability issues which arise for systems subject to communication constraints (Nair and Evans, 2004). Viewed from this perspective, attention is needed for the utilization of parallelization. There are cases in which an equilibrium may not exist where no unilateral deviations are profitable. Even if an equilibrium exists, iterations of the distributed sub-problems might not converge to this equilibrium. In this paper it is shown that for the distributed mining rate target tracking, there exists a unique equilibrium, where no unilateral deviation is profitable, and a policy iteration method is shown to converge to this equilibrium.

The remainder of this paper is organized as follows. In Section 2 the mathematical model is introduced, where each individual optimization problem is formulated as a mining rate target tracking problem. In Section 2.1, the dynamic program is solved and the closed form solutions that generate the optimal mining rate of each mine are presented. In Section 3 the distributed algorithm is given and the convergence of the algorithm toward the equilibrium is given, where profitable unilateral deviations do not exist. In Section 4, the maximum likelihood method to calibrate the stochastic price process parameters is briefly discussed and simulation results are provided. Conclusions follow.

2. Optimal target control

The mathematical model for the mining rate tracking optimization is introduced in this section. Each mine tries to track the planned extraction trajectory in order to fulfill its planned schedule. The optimization is computed on a horizon through a cost function where both deviations from the target and change in the rate of mining are penalized. Moreover, these plans are dependent on the stochastic process $p_t, t \geq 0$, the price process.