Int. J. Production Economics 157 (2014) 39–48

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal dynamic pricing and ordering decisions for perishable products

Ek Peng Chew [a], Chulung Lee [b,*], Rujing Liu [a], Ki-sung Hong [c], Anming Zhang [d]

[a] Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore
[b] School of Industrial Management Engineering and Graduate School of Management of Technology, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
[c] Graduate School of Management of Technology, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
[d] Sauder School of Business, University of British Columbia, 2053 Main Mall, Vancouver, BC, Canada V6T 1Z2

ARTICLE INFO

Article history:
Received 30 September 2012
Accepted 17 December 2013
Available online 9 January 2014

Keywords:
Inventory
Dynamic pricing
Perishable product
Demand transfer

ABSTRACT

In this paper, we determine the order quantity and the prices for a perishable product with a multiple period lifetime. Demands for products of different ages are dependent on the prices of mutually "substitutable" products. The problem for a product with lifetime of two periods is first analyzed and the stochastic dynamic programming model is developed. Given the inventory level for the old product (product of age 2), the expected profit is a concave function of the order quantity, the price of the new product (product of age 1) and the discounted price of the old product (product of age 2). The computational results show that the total profit significantly increases when demand transfers between products of different ages are considered. For a product with lifetime of longer than two periods, a heuristic based on the optimal solution for a single period problem is proposed for a multiple period problem.

1. Introduction

Companies today are facing the increasingly volatile business environments, characterized by shorter product life cycles and ever quickening technological developments. In order to achieve competitive edges, new (versions of) products must frequently be introduced to the market. In this case, product life cycles of old and new products are overlapped (see Fig. 1), and old and new products coexist in a considerable duration (Gaimon and Singhal, 1992; Dujowich and Lee, 2009).

When new versions of products enter the market, old (versions of) products may be offered at discounted prices. This discount enables a quick reduction of the inventory and is easily found in practice, such as in electronics and automobile industries. The retail price of new products as well as the discounted prices of old products must carefully be determined. If the prices for new and old products are sufficiently close, the customers may decide which products to purchase based on the prices of both products, rather than the price of the target products only. For example, a customer intending to purchase a newer version product and finding it too expensive may purchase an attractively priced older version product, instead. Thus, in order to maximize the profit, the price of a new product and the discounted prices of old products must be determined simultaneously, considering such demand transfers between the new and old products.

The integration of inventory and pricing decisions is first studied by Whitin (1955) who addresses a single period problem. Other researchers such as Zabel (1972), Thomas (1974) and Federgruen and Heching (1999) have considered a multiple period problem under different assumptions. These literatures focus on the coordinated pricing and ordering decisions for a single, non-perishable product. That is, the product quality and the product value, regardless of product ages, remain unchanged. However, due to rapid developments of new technologies, product value quickly diminishes and this "perishable effect" must be explicitly considered. Hence, it is important to differentiate a perishable product with respect to its ages. Products of different ages may capture different market segments. Additional revenue and profit can be obtained by differentiating prices for products of different ages. Furthermore, it is necessary to consider demand transfers among those products of different ages.

Some researchers such as Bassok et al. (1999) and Gerchak et al. (1996) have considered a multiple product inventory problem with substitution. However, they do not consider pricing in their model. Gallego and van Ryzin (1997) consider a multiple period pricing problem with multiple products sharing common resources. Demand for each product is a stochastic function of time and product prices. An upper bound for the expected revenue is obtained by analyzing

* Corresponding author. Tel.: +82 2 32903395; fax: +82 2 9295888.
E-mail address: leecu@korea.ac.kr (C. Lee).

0925-5273/$ - see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2013.12.022

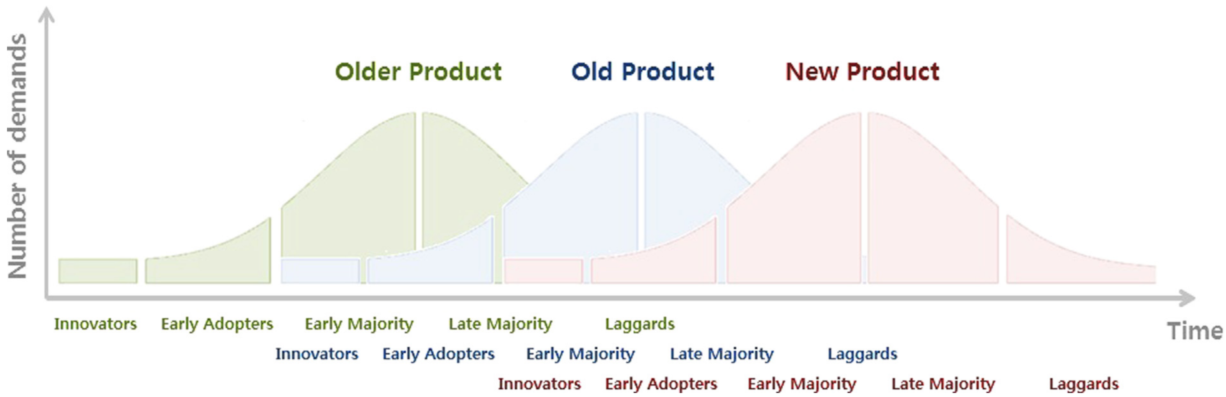

Fig. 1. Overlapping product life cycle.

this problem under the assumption of deterministic demands. The solution for the deterministic problem is employed for two heuristics for the stochastic problem that are shown to be asymptotically optimal as the expected sales volume goes to infinity. Instead of approximating the stochastic problem with a deterministic one, we efficiently solve the stochastic problem for a perishable product with lifespan of two periods. Furthermore, we determine the order quantity of the new product as well as the prices of both new and old products.

The first paper that combines pricing and capacity decisions is Birge et al. (1998), who address a single period model. By assuming the demand to be uniformly distributed, they obtain the optimal pricing and capacity decisions for two products. Maihami and Kamalabadi (2012) and Avinadav et al. (2013) also consider a single period problem. Maihami and Kamalabadi (2012) assume that demands are price and time sensitive and develop an optimization model to determine the optimal price, the optimal order quantity and the optimal replenishment schedule for non-instantaneously deteriorating items. Avinadav et al. (2013) also employ a price and time dependent demand function and develop a mathematical model to calculate the optimal price, the order quantity and the replenishment period for perishable items. In contrast, we consider a multiple period problem with a general demand distribution.

Chew et al. (2009) assume that demands for perishable products are price sensitive, and develop an optimization model to determine the price and the inventory allocation for a perishable product with predetermined multiple lifetimes. Recently, Chew et al. (2013) consider a coordinated pricing and inventory problem with a perishable product of multiple-periods lifespan. However, the demands for products of different ages are assumed to be independent from each other, and they fail to consider demand transfers among products of different ages.

In this paper, we consider a multiple period problem for a perishable product with lifetime of multiple periods that allows for substitution among products of different ages. The demands for products of different ages are dependent both on their own prices and on the prices of substitutable products, i.e., products of “neighboring ages”. The products of neighboring ages are defined by the products that are a period older or younger than the target products. A periodic review policy is used. The objective is to find the optimal prices for products of different ages and the optimal order quantity for a new product, in order to maximize the total profit over the multiple periods. The insights obtained from this study help to make pricing and ordering (or production capacity) decisions for perishable products and mass customized products (products with short life cycles) effectively and efficiently, to significantly increase the total profit.

The remainder of this paper is organized as follows: in Section 2, the assumptions and notation are provided. A product with lifetime of two or more periods is considered and the dynamic programming model for a multiple period profit maximization problem is developed. In Section 3, the model for the product with lifetime of two periods is analyzed. The optimal order quantity for the new product, its optimal price and the optimal discounted price for the old product are computed. Several structural optimality properties are also obtained. In Section 4, we present the computational results for the product with lifetime of two periods. For a product with lifetime of longer than two periods, a heuristic based on the optimal solution for a single period problem is proposed in Section 5. A brief summary is contained in Section 6.

2. Problem formulation

In this section, we consider a perishable product with an M period lifetime. Let index $i=1,\dots, M$ denote the ages of the products, where $i=1$ represents that the product is new. Hence in any period, there exist products of M different ages. The following notation is employed in this paper:

y = order quantity for a new product
x_i = inventory level for a product of age i, $i=2,\dots, M$
p_1 = retail price of a new product
p_i = discounted price for a product of age i, $i=2,\dots, M$
π_i = penalty cost for a product of age i, $i=2,\dots, M$
h = holding cost per period (regardless of ages)
c = purchasing cost for a new product
α = discounted factor per period

We assume that each aged product is purchased by a distinctive demand class. For products of age i, the price that the customers from a respective demand class (i.e., demand class i) are willing to pay is assumed to be confined in an interval $[p_i^{\min}, p_i^{\max}]$. Moreover, price intervals of demand classes are non-overlapping with $p_i > p_{i+1}$. Even though demand classes are categorized by these price intervals, we allow the customers of each class to move up or move down to neighboring demand classes, depending on the differential pricing. In particular, the demand for class i is dependent on p_{i-1}, p_i, p_{i+1} and is represented by a given linear stochastic demand function as follows:

$$\begin{aligned} t_i &= \mu_i(p_{i-1}, p_i, p_{i+1}) + \varepsilon_i \\ &= b_i - a_i p_i + l_{i+1,i} p_{i+1} + l_{i-1,i} p_{i-1} + \varepsilon_i \end{aligned} \tag{1}$$

where $b_i > -\varepsilon_i^{\min}$ and $a_i, l_{i+1,i}, l_{i-1,i} \geq 0$

$l_{10} = l_{01} = l_{M+1,M} = l_{M,M+1} = 0$