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# An improved decision rule for emergency replenishments

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This paper provides a new decision rule for emergency replenishments in an inventory system. The decision rule is a generalization of a previous decision rule suggested and evaluated in Axsäter (2003, 2007). An improvement step is added to this rule and this often means considerably better performance. The decisions are based on complete information about the system state. An advantage with our decision rule is its generality. It is possible to handle batch ordering, compound Poisson demand and emergency replenishments that take time. The decision rule has also a performance guarantee in the sense that emergency replenishments will always lead to lower expected costs. The rule can also be used in connection with lateral transshipments.

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## 1. Introduction

In this paper we consider a single-stage inventory system with compound Poisson demand and complete backordering. Normally, the inventory is replenished by a continuous review $(R, Q)$ policy. However, it is also possible to replenish from another supplier, who provides quick but expensive so-called emergency replenishments. It is rather obvious that we should use emergency replenishments when there is a shortage, or when we expect a shortage to occur relatively soon. Such replenishments are ordered in a periodic review system. There are standard inventory related costs like ordering costs, holding costs, backorder costs, and extra costs for emergency replenishments. The question is how different types of orders can be combined in an efficient way. In Axsäter (2007) essentially the same problem is considered. The suggested decision rule minimizes the expected costs under the assumption that there is only a single opportunity for an emergency replenishment. This rule is then used repeatedly as a heuristic. In Axsäter (2007) emergency replenishments are evaluated when a demand has occurred and the inventory level is negative. In the present paper we consider two decision rules. One is the decision rule in Axsäter (2007) with the modification that emergency replenishments are evaluated in a periodic review system. We denote this decision rule the old decision rule. In our new decision rule we add a certain improvement step that works well in our numerical tests.

To determine our decision rules we need complete information about the state of the considered inventory system, e.g., when outstanding orders will be delivered. See Section 3 and/or Axsäter (2007). Such information is also used in Howard et al. (2010), but in a different way. They consider a policy where a request for an emergency shipment is based on the time until an outstanding order will reach the considered stock.

A major advantage with our decision rules is their generality. We are able to handle batch ordering and compound Poisson demand. Furthermore, it may be assumed that emergency replenishments take time. Most other related papers assume that emergency replenishments take no, or essentially no, time. See e.g., Moinzadeh and Nahmias (1988), Johansen and Thorstenson (1998), Minner (2003), Huang et al. (2011), and Johansen (2012). In Johansen (2012) a periodic review Markov model is considered as an approximation of a continuous review model. The lead time for emergency orders is chosen as the period length, and it is assumed that the lead time for normal replenishments is an integer multiple of the lead time for emergency orders. These assumptions are in some settings quite restrictive. When this model is optimized by a value-iteration algorithm the numerical results are very promising. Johansen (2012) also suggests two heuristics that are similar in spirit to the technique in Axsäter (2007). One heuristic uses for normal replenishments an $(R, S)$ policy instead of the $(R, Q)$ policy used by Axsäter (2007). A heuristic that is more related to the Markov model is also suggested. It turns out that this heuristic performs a little better than the other two in the numerical tests.

Our decision rule for emergency replenishments is based on the state of the considered inventory system. The state of the inventory system will obviously change when an emergency order is triggered. When instead dealing with lateral transshipments the state changes are related but different. A lateral transshipment between two local sites will change the state both for the delivering and the receiving site. There are quite a few papers that deal with lateral transshipments. Yang et al. (2013) consider, like we do, transshipments that take time and is therefore in a sense related to this paper. Other papers dealing with lateral transshipments are e.g., Olsson (2009), Axsäter (2003), Axsäter et al. (2013), Paterson et al. (2010), and the review by Paterson

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et al. (2011). The approach in Axsäter (2003) is related to the methodology in Axsäter (2007) and the present paper.

The paper is organized as follows. In Section 2 we provide a complete problem formulation. Section 3 presents the old decision rule. The new improvement step is described in Section 4. Section 5 provides some numerical results, and Section 6 gives a few concluding remarks.

## 2. Problem formulation

A single warehouse facing compound Poisson customer demand is considered. The warehouse is normally replenished by a continuous review $(R, Q)$ policy, i.e., when the inventory position (stock on hand, plus outstanding orders, and minus backorders) declines to or below the reorder point $R$, an order is triggered. The size of the order is the minimum number of batches of size $Q$ bringing the inventory position strictly above $R$. The lead time for such normal replenishments is constant. However, the warehouse can also use periodic review emergency orders that have a shorter constant lead time. (The assumption of periodic review is discussed further below.) The shorter lead time is not necessarily zero. Emergency deliveries incur additional costs. Furthermore, we assume complete backordering. We consider standard ordering, holding and backorder costs together with other possible costs associated with emergency deliveries. There are ordering costs both for normal and emergency replenishments. Let us introduce the following basic notation:

$L$=constant lead time for normal replenishments,
$\ell$=constant lead time for emergency replenishments,
$R$=reorder point for normal replenishments,
$Q$=batch quantity for normal replenishments,
$T$=review period for emergency orders,
$q$=variable order size for an emergency order,
$\lambda$=customer arrival intensity,
$f_j$=probability for demand quantity $j$, $f_j=0$ for $j<1$,
$f_j^n$=probability that the total number of units demanded by $n$ customers is $j$, i.e., the $n$-fold convolution of $f_j$,
$\mu=\sum_{j=1}^{\infty} jf_j$=average size of a customer demand,
$A$=ordering cost per batch for normal replenishments,
$a$=ordering cost for emergency replenishments,
$h$=holding cost per unit per unit time,
$b$=backorder cost per unit per unit time,
$\delta_c$=additional cost per unit for emergency replenishments.

*Remark*: In principle, the holding cost will depend on the mix of normal and emergency replenishments, because the replenishment costs are different. As an approximation we disregard this dependence.

Let us first focus on the simple case when it is not possible to use emergency replenishments. Furthermore, we assume initially that the inventory position is kept at $k$ all the time. The expected holding and backorder costs per unit of time are denoted $C(k)$. Following Axsäter (2006) we get

$$C(k)=(h+b)e^{-\lambda L}\sum_{j=0}^{k-1}(k-j)\sum_{n=0}^{j}\frac{(\lambda L)^n}{n!}f_j^n+b(\lambda L\mu-k). \quad (1)$$

It is well-known that when applying an $(R, Q)$ policy the inventory position is uniformly distributed on the integers $[R+1, R+2, \ldots, R+Q]$. In the special case when $Q=1$, the policy can equivalently be classified as an $S$ policy with $S=R+1$. The long run holding and backorder costs $C$ per unit of time can then be obtained by averaging over these inventory positions.

$$C=\frac{1}{Q}\sum_{k=R+1}^{R+Q}C(k) \quad (2)$$

Adding the ordering costs we obtain the total costs $TC$

$$TC=\frac{A\lambda\mu}{Q}+\frac{1}{Q}\sum_{k=R+1}^{R+Q}C(k) \quad (3)$$

It is easy to optimize (3) and determine the optimal $R^*$ and $Q^*$ as well as the corresponding $TC^*$, for example by using the technique in Federgruen and Zheng (1992). Note that the obtained $R^*$ and $Q^*$ are normally only optimal without emergency replenishments. However, because the considered optimization is so simple it is often practical to use $R^*$ and $Q^*$ also in the general case where emergency replenishments are possible.

Our purpose is to determine a suitable replenishment policy for the case when emergency replenishments are possible. We shall assume that a $(R, Q)$ policy is still applied. This policy is combined with a decision rule for emergency orders. Our main focus is on the decision rule for emergency orders (given reorder point and batch quantity for normal replenishments). It is assumed that decisions concerning emergency orders can be based on complete information about the present state of the system. For example, for outstanding orders that have not yet reached the warehouse we know the remaining times until delivery.

Furthermore, it is assumed that the emergency orders are evaluated and initiated in a periodic review system with review period $T$. If, for example, $T=1$ day, we evaluate the possibility of an emergency order once every day. In Axsäter (2007) it is instead assumed that emergency orders are considered in connection with customer demands. The normal orders are, however, in both cases triggered by a continuous review system. The assumption of periodic review for emergency orders has some advantages in our context. For example, it is easy to study how important it is to evaluate emergency orders very frequently. A smaller $T$ means more frequent evaluations. Clearly, when $T\to 0$ the periodic review system for emergency orders will approach a continuous review system. When using our generalized evaluation procedure it is also natural to have a constant period between the evaluations of emergency orders. Note furthermore that by our assumptions the probability is zero that a normal order will be triggered at exactly the same time as an emergency replenishment. This is a minor technical advantage.

## 3. Old decision rule

Let us first give a complete description of the state of the considered inventory system. Assume initially that there are no emergency orders and that $R$ and $Q$ are given. The long run expected holding and backorder costs $C$ are easily obtained from Eqs. (1) and (2). However, starting at a certain time the expected future holding and backorder costs are normally not exactly $C$ because these costs are also affected by the initial state of the inventory system. There may be stock on hand or backorders. There may also be orders on their way from the outside supplier. These orders will arrive at the warehouse at certain known times. Let us introduce the notation: $x^-=\max(-x, 0)$. The state of the inventory system is completely characterized by

$$X=(IL, t_{IP}, t_{IP-1}, \ldots, t_{1-(IL)^-}) \quad (4)$$

where
$IL$=inventory level,
$IP$=inventory position. Recall that IP is simply the sum of IL and the number of outstanding orders,
$t_i$=remaining delivery time for the item that will satisfy the $i$-th future demand for a unit. We assume a first come – first served policy, so $t_i \ge t_{i-1}$.

We consider individual units. A batch of size $Q$ on its way to the warehouse is consequently represented by $Q$ units with the same

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