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A note on generalized single-vendor multi-buyer integrated inventory supply chain models with better synchronization

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ABSTRACT

In a recent paper, Hoque (2011) developed two single-vendor multi-buyer models with synchronization by transferring the vendor′s lot with equal and/or unequal sized batches. He reported that his proposed models lead to significant cost reductions when compared to the existing related ones in the literature. In this note, we show that the comparison of Hoque′s model with Zavanella and Zanoni′s (2009) model is not appropriate as they are based on different treatments of the total ordering costs. Furthermore, in case of zero transportation cost, we show that Hoque′s models lead to impractical solutions. Even when we adjust the total ordering costs to be similar to the one in Zavanella and Zanoni′ model, Hoque′s models do not generate promising results as reported with the original form of the ordering costs.

1. Introduction

A high level of coordination and cooperation among supply chain members is essential for survival in a very competitive market. Such collaboration is vital to efficiently manage inventory, deliver products to the final customer, mitigate the bullwhip effect, and respond quickly to sudden change in demand. In particular, the integrated vendor–buyers production and inventory decisions have received a lot of attention in recent years as it represents the building block for the wider supply chain. Typically, the retailers (buyers) observe a deterministic demand and order lots from the manufacturer (vendor). The vendor satisfies this downstream demand through manufacturing the requested product in lots, where each produced lot is shipped to the buyer in batches. The problem is to find the number of shipments, timing, and size of each batch such that the joint manufacturer and retailers cost is minimized.

There are several papers dealing with the integrated single-vendor multi-buyer problem. In general, the integrated solution results in a better solution than when the vender and buyers operate independently. However, the different parties will not benefit equally from the coordination. In addition to synchronizing shipments to the buyers, researchers proposed incentives in the form of discounts and various pricing schemes to make the coordinated solution attractive for both parties. Works in this line of research include Viswanathan and Piplani (2001), Bernstein et al. (2006), Li and Zhang (2008), Karabati and Sayin (2008), and among others. Authors who focused on the coordination and synchronization issues include Kim et al. (2006), Gurbuz et al. (2007), Sarmah and Goyal (2008), Hoque (2008, 2009), and Zavanella and Zanoni′s (2009). Among other issues addressed in the context of single-vendor multi-buyer include demand uncertainty (Bernstein and Federgruen, 2005), information sharing (Li and Zhang, 2008), and replenishment routing (Li et al., 2008).

Recently, Hoque (2011) proposed two single-vendor multi-buyer models with synchronization by transferring the vendor′s lot with equal and/or unequal sized batches to the buyers. He reported significant cost reductions compared to the existing related models in the literature.

In this note, we show that the comparison of Hoque′s model with Zavanella and Zanoni′s (2009) model is not appropriate as the two models are based on different functional forms of the total ordering costs. Furthermore, in the case of negelible transportation cost, we show that Hoque′s (2011) model leads to an unrealistic solution. When adjusting the total ordering costs to be similar to the ones in Zavanella and Zanoni′s model, Hoque′s model does not generate promising results as claimed in Hoque (2011) with original form of the ordering costs.

The remainder of this paper contains an analysis of Hoque′s models I and II (2011) in Section 2, modifications of some of Hoque′s models in Section 3, and some concluding remarks in Section 4.

2. Analysis of Hoque′s models I and II

Hoque′s models were formulated under a critical assumption which under-estimates the total ordering costs. It is stated on page

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466 that the set-up plus ordering plus transportation cost per cycle is given by $S'+nA$, where

$$S' = S + \sum_{i=1}^{m} S_i \quad \text{and} \quad A = \sum_{i=1}^{m} T_i$$

with S being the vendor set-up cost, S_i and T_i the ordering and transportation costs for the ith buyer, respectively.

In other words, Hoque (2011) assumed implicitly that each buyer incurs the ordering cost only once regardless of the number of orders placed and received. This is not the case for Z&Z's model (see Eq. (2) in their paper), and for all single-vendor multi-buyer models encountered in the literature (see for example, Banerjee and Burton, 1994; Lu, 1995; Yang and Wee, 2002; Chan and Kingsman, 2007; Darwish and Odah, 2010). Therefore, given that different functional forms of the total set-up and ordering costs are used in Hoque and Z&Z papers, it is inappropriate to compare the performance of the two models. More importantly, Hoque's model generates impractical solution when the transportation cost is negligible. In fact, when illustrating his model with Z&Z's numerical example having zero transportation cost, Hoque recommended the solution of case I of his second model. According to this case, one has to iterate the process of calculating the smallest vendor's batch size, z, for increasing the number of shipments, n, until the minimal cost is obtained. Following this approach, n and z will tend to infinity and zero, respectively, when the transportation cost is zero. In this case, one has to stop at the value of n when z is approximately *one unit*. Such solution is impractical, especially when the shipped quantity is indivisible, since this batch of *one unit* size has to be proportionally distributed among m buyers according to their demands. Hoque's obtained such solution when solving Z&Z's example with two buyers. Moreover, on page 466, Hoque states that "… if $T_i=0$, for all i, …all batches will be unequal. Thus the solution will be by Case II." However, when we go to case II we find that the formula for n (Eq. (5) in Hoque) is undefined since L involved a division by zero. In the following proposition, we generalize such results for any problem with zero transportation cost. We focus on cases that highlight our concerns with the models.

Proposition. *For zero transportation cost, i.e.,* $T_i=0$, *we have the following*:

(a) *Model* I—*case* II: *the total cost is strictly decreasing in n and the optimal solution is*

$$Q^* = \sqrt{\frac{2Dk(k+1)S'}{(k-1)(h+kh')}} \tag{1}$$

$$n^* = Q^*$$

$$z^* = \frac{n(k-1)}{k^n} \to 0. \tag{2}$$

(b) *Model* II—*case* I: *the optimal solution is*

$$z = y = 1,$$

$$n^* = \sqrt{\frac{2DkS'}{(k-1)h'}}, \tag{3}$$

$$Q^* = n^*$$

$$C(n^*, 1) = \frac{(h+h')}{2k} + \sqrt{\frac{2DS'(k-1)h'}{k}} \tag{4}$$

where

z=smallest vendor's transfer batch size.
y=equal-sized batch size shipped from vendor. There are $(n-l)$ equal-sized batches among the n batches transferred to the buyers. The other l batches are of sizes z, kz, k^2z, …, $k^{l-1}z$.
Q=the lot transferred from the vendor to the buyers.
$k=P/D$, P is the vendor annual production rate and D is the cumulative buyers' demand

$$h' = \frac{\sum_{i=1}^{m} D_i h_i}{D}$$

Proof.

(a) Model I—case II: under the assumption that a batch is transferred when the previous is consumed by the buyers and when $n=l$ or $y=0$ and $A=0$, Hoque's formulation of the single-vendor multi-buyers problem with synchronization is as follows:

$$\min_{n,\,Q,z} C(n,Q,z) = \frac{zh}{k} + \frac{Q(k-1)h}{2k} + \frac{z^2(k^{2n}-1)(h'-h)}{2(k^2-1)Q} + \frac{S'D}{Q} \tag{5}$$

Subject to

$$\frac{(k^n-1)}{k-1} z = Q \tag{6}$$

$$n \le Q$$

By substituting (6) in (5) to replace z we obtain

$$\min_{n,\,Q} C(n,Q) = Q\left\{\frac{(k^n+1)(k-1)(h+kh')}{2k(k^n-1)(k+1)}\right\} + \frac{S'D}{Q} \tag{7}$$

$$n \le Q$$

For a given n, the objective function in (7) is convex in Q, we have

$$Q^*(n) = \sqrt{\frac{2k(k+1)(k^n-1)S'D}{(k^n+1)(k-1)(h+kh')}} \tag{8}$$

For a given, Q, the objective function in (7) is strictly decreasing in n since

$$\frac{\partial C(n,Q)}{\partial n} = -\frac{Qk^n \ln(k)(h+kh')(k-1)}{k(k^n-1)^2(k+1)} > 0.$$

Therefore, it is optimal to have n as large as possible, but not larger than Q^*. From (8) we find

$$n^* = Q^* = \lim_{n\to\infty} Q(n) = \sqrt{\frac{2k(k+1)S'D}{(k-1)(h+kh')}} \tag{9}$$

From constraints (6) and (9) it is easy to see that (2) holds.

(b) Model II—case I: under the assumption that a vendor's batch is transferred just after its completion, Hoque formulated the single-vendor multi-buyers problem with synchronization as follows:

$$\text{Min } C = \frac{zh'}{k} + \frac{Q(k-1)h'}{2k} + \left\{\frac{z^2(k^{2l}-1)}{k^2-1} + (n-l)y^2\right\}\frac{h-h'}{2kQ} + \frac{(S'+nA)D}{Q} \tag{10}$$

Subject to

$$\frac{(k^l-1)}{k-1} z + (n-l)y = Q, \quad y = z \quad \text{if} \quad l = 1 \quad \text{or} \quad y \le k^l z \tag{11}$$

It can be shown (see Hoque, 2011) that when $A=0$, then either

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