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The routed inventory pooling problem with three non-identical retailers

Harmen W. Bouma, Ruud H. Teunter *

University of Groningen, Department of Operations, P.O. Box 800, 9700 AV Groningen, The Netherlands

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ABSTRACT

We consider a single period inventory problem with three non-identical retailers in which items can be pooled at a predetermined point in time. Since multiple items are often pooled with a single visit to each retailer, we use a predetermined (shortest) route for redistribution. We derive cost expressions for this routed pooling policy as well as for the no pooling and complete pooling policies, which serve as benchmarks. In a numerical investigation we analyze how much of the pooling benefits can be captured if the route for redistribution is fixed. Furthermore, we investigate how stock is distributed among retailers at the beginning of the period and we look into the influence of a retailer's size and position in the route on expected costs.

1. Introduction

During the past decades, both inventory control and routing problems have received considerable amounts of attention in the literature. As both inventory and transportation significantly contribute to costs in supply chains, the combination of inventory management and routing has gained popularity in recent years (Andersson et al., 2010). The majority of these combined inventory routing problems focus on order frequency and transportation routes and assume that demand rates are deterministic.

In reality, the assumption of deterministic demand rates holds very rarely. In order to compensate for strong fluctuations in demand, companies often redistribute inventory among stock points between replenishments. A specific real life case that we encountered is that of a distributor of paint in the Netherlands. This distributor periodically supplies a number of retail locations. At a predetermined point in time between replenishments, the distributor visits all locations in a fixed order and balances stock levels. Since the routing decision is not taken at item level, the distributor is able to redistribute stock levels of multiple products with a single visit to each retailer. Using a fixed rather than (SKU dependent) flexible route affects the regular replenishment levels of the retailers, as retailers visited earlier in the route cannot receive items from retailers visited later at moments of redistribution. We will refer to the problem of determining the optimal route as well as the replenishment levels as the *routed inventory pooling problem* (*RIPP*).

As will become clear from the literature review in the next section, the RIPP is new to the literature. Our exploratory research considers the RIPP with three (non-identical) retailers for a single period (between replenishments). This allows us to study the effects of fixed routing for retailers at the start, middle and end of a route whilst keeping exact (numerical) analysis tractable. Without loss of generality, we number the retailers 1, 2 and 3 in the order that they are visited when redistributing. This implies that stock may only be moved from retailer 1 to 2 and 3 and from retailer 2 to 3. We do not only derive cost distributions for the routed pooling policy, but also for the no pooling and complete pooling policies since they serve as ideal benchmarks for routed pooling. It should be noted that our single period model can be interpreted as a setting with negligible lead time, and it could therefore serve as a building block for multi-period problems.

We obtain a number of managerial insights. The optimal way to design a route is to visit small retailers with highly variable demand at the start and large retailers with stable demand at the end. An encouraging result is that optimal routed pooling solutions achieve 80–90 per cent of the savings achieved by complete pooling (i.e. pooling with complete routing flexibility at the SKU level).

The paper is organized as follows. Section 2 reviews the related existing literature. In Section 3 we give a problem definition and present the models for the no pooling, complete pooling and routed pooling case. Recall that the no pooling and complete pooling models serve as benchmarks for the routed pooling model. In Section 4 we derive analytical cost expressions, and in Section 5 we numerically determine cost-minimizing solutions for a number of instances and look into the distribution of safety stock over retailers and the effect of the size of retailers and their position in

* Corresponding author. Tel.: +31 50 363 8617.
E-mail addresses: bouma.harmen@gmail.com (H.W. Bouma), r.h.teunter@rug.nl (R.H. Teunter).

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the route on total expected costs. The final section concludes and gives directions for future research.

2. Literature review

This research is related to two main streams of the literature: inventory routing problems (IRP) and inventory pooling. These are discussed next, after which we will discuss a few other related papers.

In the field of inventory pooling, some early papers deal with redistribution at the beginning of the period (Allen, 1958, 1961, 1962), or at reorder moments (Gross, 1963; Karmarkar, 1997). These papers assume that lead times are negligible. Also in the model by Coelho et al. (2012), the lateral transshipments occur at replenishment moments. For positive lead times, Diks and De Kok (1996) consider a setting in which redistribution occurs directly after replenishment but before a new order is placed. Das (1975), Tagaras and Vlachos (2002) and Jönsson and Silver (1987) consider models in which redistribution takes place at a predetermined moment in time *during* an order cycle. Finally, Agrawal et al. (2004) present a model in which the time of redistribution is variable and in which the decision on the moment of redistribution is combined with the rebalancing decision itself. For a more comprehensive overview of the lateral transshipment literature we refer to Paterson et al. (2010).

IRP consider the combination of inventory control and routing. They have gained increasing attention in recent years and we refer interested readers to Andersson et al. (2010) for a review. However, the majority of contributions considers a setting with deterministic demand and focuses on replenishment frequency rather than reorder levels. Few papers on IRP consider uncertain demand and Coelho et al. (2012) are the only authors that study the combination of inventory pooling and routing. Coelho et al. investigate the problem where stock is redistributed at the moment of replenishment. For the replenishments they follow a standard VRP procedure in a rolling time horizon. In addition to the replenishments, deliveries from the central warehouse or lateral transshipments between pairs of retailers are allowed, but they are outsourced to a third party logistics provider. As such, the lateral transshipments are not incorporated in the routes. In the paper, heuristic methods are developed for two different order policies.

There are also a few papers from the field of lateral transshipments that are related to our study, namely those that consider restricted transshipment options (where we consider a restricted redistribution route). We do remark that the setting of these papers is somewhat different than ours, since lateral transshipments are used in reaction to shortages whereas redistribution/ pooling mainly serves to prevent them. Axsäter (2003) and Olsson (2010) both consider a situation with unidirectional lateral transshipments (ULT). Axsäter develops an approximation technique to find the order-up-to levels and compares the outcome to the model without lateral transshipments. He finds that ULT have cost benefits compared to the case without lateral transshipments. Olsson finds an improved approximation algorithm for the same problem. Kranenburg and Van Houtum (2009) consider a different form of restriction where transshipments are allowed only from a subset of warehouses. Axsäter et al. (2013) consider a variant of this problem in which this subset consists of one warehouse.

So, we are the first to study the combination of inventory pooling and routing (IPRP) with redistribution between replenishments. We provide an exact analysis for the situation with three non-identical retailers, offering insights into how to best design routes and set replenishment levels.

3. Model

We consider a single period model with three retailers, numbered 1, 2 and 3, that are (re)supplied up to S_i, $i=1,2,3$, at the start of the period. The length of the period is normalized to one. The retailers face independently but not necessarily identically normally distributed period demand D_i with mean μ_i and variance σ_i^2. Although the normal distribution allows for negative demand, this may only cause problems for certain parameter settings, which are excluded from the experiments in Section 5. Somewhere during the period at time α, $0 \le \alpha \le 1$, the residual stock levels R_i are observed and stock may be redistributed among retailers. We therefore consider two stages, one before and one after pooling. First stage demand at retailer i is normally distributed with mean $\mu_{i1}=\alpha\mu_i$ and variance $\sigma_{i1}^2=\alpha\sigma_i^2$, while second stage demand is normally distributed with mean $\mu_{i2}=(1-\alpha)\mu_i$ and variance $\sigma_{i2}^2=(1-\alpha)\sigma_i^2$. Hence, the demand distributions for both stages are independent. It should be noted that this may not always be the case in reality. As a consequence, one should be careful with extending the results of this paper to cases in which the per stage demand distributions are correlated, since different redistribution policies may be needed. We can only use the fixed route $1\rightarrow2\rightarrow3$ for redistribution, i.e. we can only move inventory from retailer 1 to retailers 2 and/or 3 and from retailer 2 to retailer 3. This is a valid assumption as often multiple items are pooled along the same route in practice.

Although the retailers are non-identical in terms of demand, they share the same cost structure. A backorder cost b is incurred for each demand that cannot be satisfied immediately and a holding cost h for each item that remains unused at the end of the period. Backorders that occur during the first stage are satisfied at time α by residual stock of other retailers if possible. Note that these assumptions of non-identical demand but identical costs apply to the discussed real-life case and in general when retailers belong to the same company.

Recall that route decisions are typically not taken at the individual SKU level, but coordinated across SKUs to limit transportation costs, which motivates our study of a fixed route at the SKU level. Nevertheless, we assume that the transportation times are negligible. Although this may seem like a paradox at first, one should realize that the transportation times should be interpreted relative to the time between replenishments. For the case of the paint distributor described in the Introduction, replenishments are performed monthly whereas redistribution (in the Northern region of The Netherlands) takes less than a day. More general, many firms set relatively large replenishment intervals in order to limit fixed ordering and handling costs, and/or satisfy minimum order size restrictions.

4. Analysis

The main focus is on the *routed* (*inventory*) *pooling* policy. Nevertheless, we will first look into the *no pooling* and *complete pooling* policies. In the latter policy, pooling is allowed without any restrictions on the order in which retailers are visited. The no pooling and complete pooling policies serve as benchmarks for testing the performance of the routed pooling policy as complete pooling is more flexible than routed pooling whereas no pooling does not allow for any flexibility. While the number of retailers in the routed pooling case is restricted to three, we consider the general case with any number of retailers N in case of no pooling and complete pooling. In the remainder, we make use of the notation in Table 1.

4.1. No pooling

Without pooling, the problem decomposes into N identical single period newsvendor problems. Using standard results (see e.