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Optimal policy for an inventory system with power demand, backlogged shortages and production rate proportional to demand rate

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ABSTRACT

This article analyzes an inventory system for items with time-varying demand. The way by which demand occurs during the inventory cycle follows a power demand pattern. The production of items allows to add stock in the inventory during a replenishment period. The production rate is proportional to demand rate. Shortages are allowed and completely backlogged. Holding cost, shortage cost and order cost are the costs considered in the inventory model. The aim consists of the minimization of the total cost per inventory cycle. An efficient approach is developed to obtain the optimal scheduling period and the optimal reorder point. In addition, the economic lot size and the minimum cost of the inventory management is determined. Some numerical examples are presented for illustrating the proposed inventory model.

1. Introduction

Inventory models give a variety of insights to find the optimal way to manage inventories of items. The classical economic order quantity (EOQ) model is one of the most popular and useful in inventory management. That model describes the trade-off between the holding cost and the order cost, deriving an economic order quantity that minimizes the inventory total cost per scheduling period. Since Harris (1913) presented the famous economic order quantity formula, many researchers have analyzed different inventory systems relaxing some of the assumptions of the basic model. The economic order quantity model assumes that the demand of items is constant. However, this assumption is not very common in practice, and could be more realistic to consider that demand depends on time.

The study of inventory models where it is allowed that demand rate varies with time has extended the field of inventory control and it has engaged the attention of researchers in the last decades. Thus, Resh et al. (1976) analyzed deterministic lot size systems with time-proportional demand. Donalson (1977) considered the situation of linearly time-varying demand and established an algorithm to determine the optimal policy. Barbosa and Friedman (1978) presented a continuous time inventory model with time-varying demand. Silver (1979) developed an approximate solution approach to reduce the computational effort needed in the procedure proposed by Donaldson. Dave and Patel (1981) studied a inventory model for deteriorating items with time proportional demand. Ritchie (1984) studied an inventory model with linear increasing demand. Mitra et al. (1984) presented a near-optimal procedure for adjusting the economic order quantity model for the cases of increasing or decreasing linear trend demand patterns. Goswani and Chaudhuri (1991) discussed an EOQ model with shortages, assuming a linear trend in demand and finite replenishment rate. Bose et al. (1995) developed an EOQ model with a linear positive trend, allowing backlogged shortages and deteriorating items. Chakrabarti and Chaudhuri (1997) developed an economic order quantity model that focused on time-varying demand. Zhao et al. (2001) analyzed the replenishment problem with decreasing demand, developing a new approach for that problem. Wen-Yang et al. (2002) presented a model for obtaining the exact optimal solution of inventory replenishment policy problem assuming a linear trend in demand. Wu (2002) discussed also an EOQ model with time-varying demand, considering deterioration and shortages. Goyal and Giri (2003) proposed an approach for determining replenishment intervals of an inventory item with linear decreasing demand rate. Yang et al. (2004) developed a procedure for replenishment of products with non-linear decreasing demand. Sakaguchi (2009) presented a model for a multiperiod inventory system with time-varying demand rate. Omar and Yeo (2009) studied a production system

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that satisfied a continuous time-varying demand for a finished product over a known and finite planning horizon. Mishra and Singh (2010) presented an inventory model for items with constant rate of deterioration and time dependent demand. Hsueh (2011) studied inventory policies for a manufacturing system with four phases of the product life cycle: introduction, growth, maturity, and decline. The mean demand rate of products varies according to the time in the product life cycle and is linear time dependent in the growth and decline periods. Maihami and Kamalabadi (2012) developed an inventory model for non-instantaneous deteriorating items, adopting a price and time dependent demand function. Pando et al. (2012) studied an profit maximization inventory model where the unit holding cost is a function of storage-time and the demand rate depends on stock-level. Recently, Pando et al. (2013) analyzed an economic lot-size inventory model with non-linear holding cost and dependent on time and quantity, assuming that demand rate depends on the stock level and, as consequence, varies with time.

The assumptions made about demand are decisive in the inventory systems. These usually determine the complexity of inventory management models and their utility in making business decisions. The different ways by which demand occurs during a period is referred to as demand patterns. Researchers have discussed different demand patterns fitting the stage of product cycle. Usually, the demand pattern is uniform and the demand rate is constant during all the period. However, there are other ways by which quantities are taken out of inventory to fill the customers' demand. One of these ways to remove items from the inventory is the called power demand pattern, which was discussed in Naddor (1966). That pattern allows to model the case of uniform demand and represents also other situations, where either a larger portion of demand occurs toward the beginning of the period or demand is mainly concentrated at the end of the period.

The power demand pattern is mainly observed for those products whose demand is not distributed uniformly through on the inventory cycle. We can identify real-life products demands that reflect situations where demand is higher at the beginning of the scheduling period. Thus, it occurs for cooked goods such as breads, cakes, sweets, pastries, prepared food, etc., because consumers want food products that was just finished. This type of demand can also occur in fish, fresh meat, fruits, vegetables, yogurts, etc., because sales are reduced when the expiry date approaches. However, there are other goods where the demand is higher at the end of the inventory cycle. This situation occurs for products like petrol, where demand increases when the good becomes scarce. Indispensable household items can fall into this category, such as oil, coffee, sugar, water, etc. Increases in their demand occur when the amount of inventory displayed starts to decrease because of its daily use.

From the point of view of business decisions, incorporating potential demand pattern may have interesting implications for management and inventory control within organizations. The analysis considering this demand pattern can improve the effectiveness and efficiency of different economic activities, determining the optimal policy and reducing management costs.

There are several papers where demand follows a power pattern. Thus, Naddor (1966) analyzed an order-level system with power demand pattern. Goel and Aggarwal (1981) developed an order-level inventory model with power demand for deteriorating items. Datta and Pal (1988) studied an inventory system with power demand and variable rate of deterioration. Lee and Wu (2002) presented an EOQ model assuming deterioration, shortages and power demand pattern. Dye (2004) extended the Lee an Wu' model to a general class with time-proportional backlogging rate. Singh et al. (2009) developed an economic order quantity model for perishable items with power demand pattern and partial backlogging. Rajeswari and Vanjikkodi (2011) studied an inventory model for deteriorating items where shortages are partially backlogged and demand follows a power pattern. Rajeswari and Vanjikkodi (2012) analyzed an inventory model for Weibull deteriorating items with power pattern time dependent demand. Mishra and Singh (2013) presented an EOQ model for perishable items with power demand in which shortages are allowed and partially backlogged.

In the above models where the power demand pattern is considered, the length of the inventory cycle is fixed. Thus, the average total cost per inventory cycle is a function that is depending on a variable. However, in the current work, the scheduling period is not fixed and the total cost per inventory cycle depends on two decision variables: the reorder point and the scheduling period. It implies that the solving of the inventory problem is a bit more difficult. Therefore, we will study replenishment policies in inventory systems with a power demand pattern and assuming a production rate proportional to demand rate. A mathematical formulation of the inventory system is presented where the different costs considered in the inventory model are specified. The objective of the inventory problem consists of determining the optimal scheduling period, the economic lot-size, the optimal reorder point and the minimum total inventory cost per unit time.

The rest of the paper is organized as follows. Section 2 introduces the assumptions and notation used throughout the paper. Section 3 presents the analysis of the inventory system. First, the average amount carried in inventory and the average shortage during the inventory cycle are calculated. Next, the costs associated with the inventory management are determined. Section 4 proposes a procedure to calculate the optimal solution for the inventory problem. Some special cases are analyzed in Section 5. Numerical examples are provided in Section 6 to illustrate the features of the proposed model. Finally, we present conclusions and some suggestions for future research.

2. Assumptions and notation

The inventory system will be assumed to have the following properties:

The planning horizon is infinite.
The behavior of the inventory system during the scheduling period T or length of each inventory cycle is assumed to be repeated continuously.
A single item is considered over the inventory cycle.
During the scheduling period T the demand size is denoted by d. Average demand is deterministic at a rate of $r=d/T$ quantity units per inventory cycle, but the way by which quantities are taken out of inventory depends on time when they are withdrawn. This way by which demand occurs during a period will be referred to as demand pattern.
The demand rate at time t follows a power demand rate and it is expressed as $D(t)=rt^{(1-n)/n}/nT^{(1-n)/n}$, with $0 \le t \le T$. Thus, the demand up to time t $(0 \le t \le T)$ is varying with time and is assumed to be $d(t/T)^{1/n}$, where n is the demand pattern index, with $0 < n < \infty$. Fig. 1 illustrates such power demand pattern for different values of the index n.
The inventory should be replenished whenever the inventory level is equal to or below s quantity units (s is the reorder point).
The replenishment quantity, production quantity or lot size Q is the quantity to be added to inventory.
The lead time is zero or negligible.