



# Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity

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## ABSTRACT

Trade credit financing is increasingly recognized as an important strategy to increase profitability in Inventory Management. We revisit an economic order quantity model under conditionally permissible delay in payments, in which the supplier offers the retailer a fully permissible delay of  $M$  periods (i.e., there is no interest charge until  $M$ ) if the retailer orders more than or equal to a predetermined quantity  $W$ . However, if the retailer's order quantity is less than  $W$ , then the retailer must make a partial payment to the supplier, and enjoy a permissible delay of  $M$  periods for the remaining balance. In this paper, we extend the mentioned EOQ under conditionally permissible delay in payments to complement some shortcomings of the model. By contrast to the differential calculus method, we propose a simple arithmetic–geometric method to solve the problem. Furthermore, we establish some discrimination terms to identify the unique optimal solution among three alternatives, and explain those theoretical results by simply economical interpretations. Finally, we solve several numerical examples to illustrate the theoretical results and obtain some managerial implications.

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## 1. Introduction

The effect of the trade credit on the lot sizes has been studied in various researches. In practice, the supplier usually offers to the retailer a permissible delay in payments. During the permissible delay period, there is no interest charge. Hence, the retailer can earn the interest from sales revenue. However, if the payment is not paid in full by the end of the permissible delay period, then the supplier charges to the retailer an interest on the unpaid amount of the purchasing cost. It is important to remark that the permissible delay in payments produces two benefits to the supplier: (1) It should attract new customers who consider it to be a type of price reduction, and (2) It should avoid lasting price competition. On the other hand, the policy of granting a permissible delay adds not only an additional cost (i.e., the opportunity cost for receiving the purchasing amount at the end of the permissible delay, instead of immediately) but also an additional dimension of default risk (i.e., the event in which the buyer will be unable to pay off its debt obligations) to the supplier because the longer the permissible delay, the higher the default risk.

In 1913, the economic order quantity (EOQ) was proposed by Harris (1913). Since then a huge of extensions of the EOQ inventory model have been developed by researchers. A classical paper by Goyal (1985) develops an EOQ inventory model for the buyer when the seller offers a permissible delay in payments. Goyal (1985)'s model can be considered as one of seminal works in trade credit field because a stream of researches have been emerging from it in recent years. Later, Shah (1993) considers a stochastic inventory model when delays in payments are permissible. Subsequently, Jamal et al. (1997) extends Goyal (1985)'s model to allow shortages. At the same time, Hwang and Shinn (1997) add the pricing strategy to the model, and derive jointly the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Moreover, it should be noted that Teng (2002) provides an alternative conclusion from Goyal (1985), and proves that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. One year later, Chang et al. (2003) develop an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Simultaneously, Huang (2003) extends Goyal (1985)'s model to develop an EOQ model in which the supplier offers the retailer the up-stream trade credit period  $M$ , and the retailer in turn provides the down-stream trade credit period  $N$  (with  $N \leq M$ ) to his/her customers. However, Huang

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(2003)'s model has some flaws. In this context, Teng and Goyal (2007) complement the shortcoming of Huang (2003)'s model and propose a generalized formulation. Further, Teng (2009a) establishes an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. After, Chang et al. (2010) present the optimal manufacturer's replenishment policies in a supply chain with up-stream and down-stream trade credits. In a recent study, Teng et al. (2011) obtain the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. In contrast to previous mentioned inventory models, which implicitly assume that whole lot is delivered to the retailer completely perfect, Lin et al. (2012) propose an integrated supplier–retailer inventory model in which both the supplier and the retailer adopt trade credit policies, and the retailer receives some defective items. While Cheng et al. (2012) consider different financial environments when the supplier provides a permissible delay in payments. Recently, Jaggi et al. (2013) develop a new inventory model for imperfect quality items with allowable shortages under permissible delay in payments. Many related articles can be found in Chang et al. (2008), Goyal et al. (2007), Huang and Hsu (2008), Kreng and Tan (2010, 2011), Liao (2008), Ouyang et al. (2005, 2006), Shinn and Hwang (2003), Soni and Shah (2008), Teng and Lou (2012), and their references.

Huang (2007) the first proponent proposes an EOQ model under conditionally permissible delay in payments in which the supplier offers the retailer a fully permissible delay of  $M$  periods if the retailer orders more than or equal to a predetermined quantity  $W$ . However, if the retailer orders less than  $W$ , then the retailer must pay a portion of the total purchasing cost to the supplier immediately meanwhile enjoy a permissible delay of  $M$  periods for the remaining balance. The model is realistic and relevant. However, some of his mathematical expressions and graphical figures on both the interest earned and the interest charged are inappropriate. In this paper, we extend Huang (2007)'s model to complement some shortcomings of the model. In contrast to the differential calculus method, we also propose an easy-to-understand and simple-to-apply arithmetic–geometric method to solve the inventory problem. Furthermore, we establish some discrimination terms to identify the unique optimal solution among three alternatives, and explain those theoretical results by simply economical interpretations. Finally, we run computer programs for several numerical examples to illustrate the theoretical results and obtain some managerial implications.

## 2. Notation and assumption

To develop the EOQ model with conditionally permissible delay in payments, we use the following notation throughout this paper.

$D$	the annual demand rate in units
$A$	the ordering cost in dollars per order
$W$	the pre-determined quantity in units by the supplier at which the fully permissible delay in payments is granted
$c$	the unit purchase cost in dollars
$p$	the unit selling price in dollars, which is greater than or equal to the unit purchase cost $c$
$h$	the holding cost in dollars per unit per year excluding interest charges.
$I_e$	the retailer's investment return rate (or interest earned) per dollar per year
$I_k$	the supplier's interest charged to the retailer per dollar per year
$M$	the length of the permissible delay in years offered by the supplier

$\alpha$	the percentage of the purchase amount is granted the permissible delay in payments by the supplier
$T$	the retailer's replenishment cycle time in years
$Q$	the retailer's order quantity in units
$T_w$	the time interval that $W$ units are depleted to zero due to annual demand rate $D$ , hence $T_w = W/D$
$TRC(T)$	the retailer's annual total relevant cost in dollars, which is a function of $T$
$T^*$	the retailer's optimal replenishment cycle time of $TRC(T)$
$Q^*$	the retailer's optimal order quantity, which is $DT^*$

Likewise, we adopt the following assumptions in developing our proposed model as shown below:

1. If the retailer's order quantity  $Q \geq W$  (i.e.,  $T \geq T_w$ ), then fully permissible delay in payments is granted by the supplier. Hence, the retailer is allowed to pay the total purchase amount  $cQ$  at the end of the permissible delay  $M$ . If  $Q < W$ , then partial permissible delay in payment is granted, in which the retailer must immediately make a partial payment of  $(1-\alpha)cQ$ , and then must pay off the remaining balance of  $\alpha cQ$  at the end of the permissible delay  $M$ .
2. The supplier charges the retailer the prime rate  $I_k$  on unpaid balance after the permissible delay. On the other hand, the retailer may invest sales revenue into stock markets or to develop new products, and get a return rate on investment  $I_e$  during the permissible delay period.
3. Demand rate is known and constant.
4. Replenishments are instantaneous.
5. Shortages are not allowed.
6. Only one type of product is considered.
7. The time horizon is infinite.
8. While the account is not settled, the revenue is placed in an interest bank account in order to earn interests.

Now, we are ready to build up the mathematical model for the retailer to obtain its optimal order quantity.

## 3. Mathematical formulation

The annual total relevant cost  $TRC(T)$  based on  $M$  and  $T_w$  consists of the following two possible cases: (1)  $T_w \leq M$ , and (2)  $T_w > M$ . We discuss them separately below.

### 3.1. Case 1: $T_w \leq M$

- (a) The annual ordering cost is  $A/T$ .
- (b) The annual holding cost excluding interest charges is  $hDT/2$ .
- (c) The annual opportunity cost of capital has three possible sub-cases because the replenishment cycle time  $T$  may lie in three possible positions between  $T_w$  and  $M$ .

*Sub-case 1.1:  $T_w \leq M < T$ .* Since  $T_w \leq M < T$ , the retailer receives the fully permissible delay as shown in Fig. 1. The interest earned per cycle is the return rate  $I_e$  multiplied by the area of the triangle  $OVM$  (i.e.,  $pI_eDM^2/2$ ). Therefore, the annual interest earned is  $pI_eDM^2/2T$ . Likewise, the interest charged is the interest rate  $I_k$  multiplied by the area of the triangle  $MXT$  (i.e.,  $cI_kD(T-M)^2/2$ ). Hence, the annual interest charged is  $cI_kD(T-M)^2/2T$ , and the annual opportunity cost of capital is

$$\frac{cI_kD(T-M)^2 - pI_eDM^2}{2T}. \quad (1)$$

*Sub-case 1.2:  $T_w \leq T \leq M$ .* Since  $T_w \leq T$ , the retailer receives the fully permissible delay as shown in Fig. 2. There is no interest

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