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Analysis of batch ordering inventory models with setup cost and capacity constraint

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ABSTRACT

Stochastic periodic-review batch ordering inventory problems appear in many industrial settings. However, few literature deals with the optimal ordering polices for such problems, no mention to the inclusion of the fixed ordering cost and the production capacity. In this paper, we consider a single-item periodic-review batch ordering inventory system with the consideration of the setup cost and the capacity constraint for each order over a finite planning horizon. By proposing several new convex notions, we show that a batch-based (s,S) policy is optimal for the unlimited ordering capacity case, while for the limited ordering capacity case, a modified (r,Q) policy is optimal for the setting with zero ordering setup cost, and a batch-based X–Y band policy for the setting with positive ordering setup cost. Moreover, we analytically study the sensitivity of the policy parameters with respect to the capacity and batch order size, and derive the bounds on the optimal policy parameters. We further extend our analysis to the infinite horizon setting and show that the structure of the optimal policy remains similar. Finally, the numerical experiments provide some insights into the impact of model parameters on the benefit of reducing the batch size and increasing the ordering capacity, and indicate that ignoring batch requirement may lead to a significant cost increment.

1. Introduction

Traditional research on inventory management mainly focuses on studying ordering policies for continuous order size, i.e., the ordering quantity is infinitely dividable. However, in practical operations, e.g., materials usually flow at fixed batch sizes. For example, consumer packaged goods typically arrive at retailing stores in casepacks (Ketzenberg et al., 2000), finished goods may be transported in full containers from manufacturers to distributors, and work-in-process (WIP) is often processed in some convenient lot sizes between production stages. Despite that, inventory models with batch ordering models are still relatively understudied, as those models cannot produce nicely structured policies as compared to their continuous counterparties.

Moreover, the complexities in practice go far beyond the ordering batch size. Two other common factors, i.e., setup cost and capacity constraint, further complicate the ordering decisions in the production–inventory systems. The ordering setup cost is also known as the fixed cost as opposed to the variable cost. Taking the retailing store for example, the setup cost may include the search cost for a counter party, the cost for paperwork, the cost of transportation, etc. The ordering capacity is due to shortage of capital or limited resource of the retailer, or the production capacity of the supplier. In practice, the ordering capacity always stays stable or changes only slightly over time, as it normally takes a significant amount of time for suppliers to adjust his capacity. Thus, the ordering capacity can be (or approximately) viewed as a constant and observable. Although inventory models dealing with the two issues (i.e., setup cost and capacity constraint) independently are extensive, those considering these two issues together are scarce, not to mention inventory models considering setup cost, capacity constraint and batch order size at the same time. This paper seeks to fill the gap in the literature by studying the optimal policies for a capacitated inventory system with batch ordering and setup cost.

Specifically, in this paper, we consider a single-item periodic-review capacitated inventory system with batch ordering and setup cost over a finite planning horizon. At the beginning of each period, the firm first observes his initial inventory level, and then decides the ordering quantity, which should be in batch size and is constrained by the ordering capacity, to raise the inventory level so as to satisfy the demand at the end of this period. Each order will occur at a setup cost with its leadtime equal to zero. After demand realizes, the excess inventory will be taken over to the next period, while the unsatisfied demand will be backlogged.

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We derive the optimal ordering policies to minimize the total expected cost over the whole planning horizon. By proposing several new convex notions, we show that a batch-based (s,S) policy, i.e., $(\mathbf{s},\mathbf{S})_Q$ policy, is optimal when the ordering capacity is unlimited, while when the ordering capacity is limited, a modified (r,Q) policy is optimal for the case with zero setup cost and a batch-based X–Y band policy, i.e., $[\mathbf{X}-\mathbf{Y}]_Q$ policy, is optimal for the case with positive setup cost. Moreover, we analytically investigate the sensitivity of the policy parameters with respect to the capacity and batch order size, derive the bounds on the optimal policy parameters, and numerically study the value of reducing batch size and expending system capacity, investigate the loss by ignoring batch size to constitute the managerial contribution of this paper.

1.1. Literature review

Three streams of the literature in operations management are related to this paper, namely inventory models with batch ordering, inventory models with setup cost and capacitated inventory models.

Batch ordering models constitute the most relevant stream of research to our paper. In a single-stage model, Veinott (1965) shows the optimal inventory policy to be of the (r,Q) type: when the inventory level falls below the reorder point r, the optimal policy is to increase it to a position between the reorder point r and the level $r+Q$, by placing an order of a single batch (Q) or multiple batches. Chen (2000) further extends Veinott's analysis to multi-echelon systems with batch ordering and establishes the optimality of the echelon-stock based (r,Q) policy under long-run average criterion. Chao and Zhou (2009) study a serial system with batch ordering and fixed replenishment intervals (each of which may contain multiple review periods) and develop structural properties of the system. Their paper generalizes Chen (2000). Huh and Janakiraman (2012) extend Veinott (1965) and Chen (2000)'s results to the non-stationary system by employing the concept of Q-difference increasing, which is first used by Gallego and Toktay (2004). Yang et al.(2013) apply the same concept, which they re-term as "strong Q-jump-convexity", to analyze the pricing-inventory models and they also extend this concept to the case with fixed ordering cost. However, the capacity constraint has not been incorporated.

The second stream of the literature related to our model is the multi-period inventory models with the presence of setup cost. The literature in this stream is vast. One of the ancestors of these models is the Economic Order Quantity (EOQ) model of Harris (1913), which considers a continuous time model with a deterministic demand and an infinite time horizon. Arrow et al. (1951) is the first publication that studies multi-period models with stochastic demand and setup cost. They focus on finding the best policy among the class of (s,S) policies. Scarf (1960) and Veinott (1966) further analyze this problem when the excess demand is backlogged in each period and show for the first time that the (s,S) policy is optimal under different conditions. The (s,S) policy is operated as follows: if the initial inventory level is lower than the reorder point s, the optimal order-up-to level is S. Recent developments in this stream are mainly on the inclusion of multiple supply sources (e.g., Sethi et al., 2003; Fox et al., 2006), stochastic ordering cost (e.g., Chen et al., 2013 and reference therein), different ordering cost structure (e.g., Caliskan-Demirag et al., 2012; Toy and Berk, 2013), etc. Refer to Glock (2012) for the literature related to joint economic lot size (JELS) models. However, these works are established on the assumption that there is no restriction on order size.

The literature on inventory models with capacity is also related to our work. Federgruen and Zipkin (1986a,b) have shown that if there is no setup cost incurred, then the optimal policy is a modified base-stock policy which is described as follows: when the inventory level falls below a critical number S, order enough to bring total stock up to S or as close to it as possible; otherwise do not order. Their results are also verified by Kapuscinski and Tayur (1998) through a simple approach. Ozer and Wei (2004) consider a periodic-review capacitated inventory system with advance selling. They establish the optimality of the state-dependent modified base-stock policy and characterize its behavior with respect to capacity, advance demand information, and the planning horizon. In the presence of the fixed ordering cost, Shaoxiang and Lambrecht (1996) provide a counter example in which the optimal policy is no longer a modified (s,S) policy and show that the optimal policy does exhibit a systematic pattern of what is called X–Y band: if the inventory level drops below X, order full capacity; if the inventory level is above Y, do nothing; if the inventory level is between X and Y, however, the ordering pattern is not defined by the policy. Gallego and Scheller-Wolf (2000) report that further characterization of the structure within the X and Y boundaries is possible, though not very simple. Shaoxiang (2004) further extend the results to the infinite horizon case and show that the width of X–Y band is no more than one capacity, i.e., $Y-X\le C$. Recently, Duan and Liao (2013) demonstrate the performance of both decentralized and centralized controls in the simulation-based optimization setting for a capacitated single-distributor-multi-retailer supply chain system. For the capacity planning models, refer to Ho and Fang (2013), Pimentel et al. (2013) and the literatures therein. However, these works assume that the order size is infinitely divisible.

1.2. Our contribution

Compared with the existing literatures, one of our main contributions is to establish the optimal ordering policies for three models: the uncapacitated inventory models with batch ordering and setup cost, the capacitated inventory models with batch ordering and the capacitated inventory model with both batch ordering and setup cost. Table 1 lists the relationship between our contributions and some existing results.

This paper also contributes in providing new technical tools from the technique perspective. That is, we propose new convexity notions such as Q-jump-K-convexity and Q-jump-(C,K)-convexity, which extend K-convexity and (C,K)-convexity to a more general form that depends on the batch size. We also derive some properties of these notions, e.g., we show their preservation under the dynamic programming recursions. These notions can serve not only the models stated in this paper, but also the future research dealing with inventory optimization and other problems in operations management.

The rest of this paper is organized as follows. In Section 2, we provide the dynamic programming formulation of the inventory problem along with the notations used throughout the paper. In Section 3, we analyze the system without ordering capacity, for which a so-called $(s,S)_Q$ policy is proved to be optimal. Section 4 further extends the results to the situation with the finite ordering capacity. We derive the optimal ordering policies for both the zero and positive setup cost by proposing new convexity notions. In Section 5, we present some results from our comprehensive numerical experiments. Finally, we conclude the paper in Section 6. All the proofs are provided in the appendix.

2. Model formulation

Consider a firm that manages a periodic-review inventory system over a finite horizon with T periods. We index each period