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Optimal ordering and pricing policy for demand functions that are separable into price and inventory age

Tal Avinadav [a,*], Avi Herbon [a], Uriel Spiegel [a,b]

[a] Department of Management, Bar-Ilan University, Ramat-Gan, Israel
[b] Department of Economics, University of Pennsylvania, Philadelphia, USA

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ABSTRACT

We formulate and analyze two models for determining the optimal pricing, order quantity and replenishment period for items whose demand function is separable into components of price and inventory age. The first model assumes a multiplicative demand function. We provide conditions, which are satisfied by most common price-dependent demand functions, to reduce the three-variable profit maximization problem into a single-variable problem, which can be solved using an efficient line-search method. Next, we show that a genuine additive model cannot exist, and instead suggest and analyze a pseudo-additive model. However, this model is more limited than the multiplicative model in its ability to incorporate various combinations of price and inventory age effects, and reduction of the maximization problem into a single-variable problem is more complicated, except in the case of a linear price effect, which is further analyzed. For both models, we show that the optimal solution satisfies the first-order condition for equilibrium under a monopoly, with a modification that includes inventory holding costs. We solve numerical examples to illustrate the solution procedures.

1. Introduction

Most inventory management policies involve two decision variables: order quantity and replenishment period. These variables directly affect the inventory level trajectory. For some time, the literature has emphasized the unit selling price as a decision variable that affects the inventory level indirectly, through the demand function. Only recently have researchers begun considering how the demand function is influenced by the inventory age (i.e., the elapsed time measured from the most recent replenishment) in conjunction with the selling price. This is especially important for inventory modeling of perishable items, whose demand might be affected by their freshness (see, for example, You, 2005; Tsao and Sheen, 2008; Avinadav and Arponen, 2009; Valliathal and Uthayakumar, 2011; Maihami and Kamalabadi, 2012; Avinadav et al., 2013).

In this paper we optimize the order quantity, replenishment period and unit selling price to maximize profits, given a demand function that is affected by both price and inventory age. Most of the results pertain to cases in which the demand function can be separated into the latter two factors, taking either a multiplicative form or an additive form. The contribution of this study to the literature is threefold: (i) showing that under common conditions the multiplicative form maximization problem can be reduced into a single-variable problem (the replenishment period), which can be solved using the efficient line-search methods; (ii) showing that a genuine additive demand function cannot exist and suggesting and analyzing a pseudo-additive model instead; and (iii) providing economic interpretations for the optimality conditions.

We assume that demand rate (demand per unit of time) is non-increasing in inventory age, and that unit retail price is fixed. These assumptions suit inventory systems of perishable items – i.e., items that physically deteriorate or whose quality decreases over time – in stores, such as supermarkets, that replenish items before expiration but avoid discounts for reduced freshness. Examples of perishable items are food products and beverages (e.g., fresh vegetables and fruit, dairy and meat products), fashion goods, ink cartridges for printers, and batteries.

In general, the age of inventory is likely to negatively affect the demand rate for perishable items. Sarker et al. (1997) claim that this effect occurs because consumers tend to feel less confident purchasing perishable items as their expiration dates grow nearer. Disregarding the effect of perishable products' limited shelf-life on their demand may lead to significant losses. For example, according to van Donselaar and Broekmeulen (2012), the United States Department of Agriculture (USDA) estimates that average annual food losses due to leftover inventory in supermarkets in 2005 and 2006 were 11.4% for fresh fruit, 9.7% for fresh vegetables and 4.5% for fresh meat, poultry and seafood.

In the next section we present the literature dealing with the influence of price and time on demand rate. In Section 3, we explain

* Corresponding author. Tel./fax: +972 3 5318143.
E-mail address: tal.avinadav@biu.ac.il (T. Avinadav).

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the approach we use to find the optimal inventory policy and to evaluate its sensitivity to changes in the decision variables. We then formulate the model and present general properties of the optimal policy. In Section 4, we thoroughly analyze demand functions that take a multiplicative form (i.e., functions in which the components of price and inventory age are multiplied by each other), and in Section 5, we analyze demand functions that take an additive form and discuss their limitations. Each analysis is followed by numerical examples that demonstrate the procedure for obtaining the corresponding optimal solution. We conclude with the main findings and directions for future work.

2. Literature review

The literature on the effects of price on demand in inventory models is comprehensive and well developed (see a literature review in Petruzzi and Dada (1999); more recently, Chang (2013) proposed an economic lot size model for price-dependent demand under quantity and price discount). Most studies in this stream of literature assume that demand is a decreasing function of price (see examples of common convex and concave demand functions in Kocabıyıkoğlu and Popescu, 2011).

Nasr et al. (2013) and Hsu and Hsu (2013) discuss the effect of items' quality – specifically, the presence of defective items – on the optimal order quantity; however, they do not consider the negative effect of time on product quality. Much of the literature on the effects of time on demand focuses on the additional depletion of inventory (beyond depletion resulting from sales to consumers) due to deterioration over time, or to declining/growing markets. A survey regarding different types of deteriorating inventory appears in Raafat (1991), and a review of subsequent developments in this field is presented in Goyal and Giri (2001). Some models assume that inventory deterioration is a stochastic process (e.g., Philip, 1974; Tadikamalla, 1978), whereas others assume that it is deterministic (e.g., Wu et al., 2006; Musa and Sani, 2012; Taleizadeh et al., 2013). Nahmias et al. (2004) assume that prior to the inventory's expiration, demand is not affected by the age of the products, whereas Avinadav and Arponen (2009) assume that the demand rate increases in the remaining shelf-life, owing to consumers' preference for fresh items.

Most models that consider the negative effect of elapsed time, measured from the most recent replenishment, on demand rate do so implicitly. For example, in models that assume that demand is dependent on inventory level (see a review by Urban, 2005 and a recent study by Pando et al., 2013), solving the differential equation of the inventory level yields a demand rate that diminishes over time (within the period). In general, existing modeling approaches that assume an inventory-level-dependent demand rate are not applicable to inventory systems of perishable items, owing to the following two reasons: First, many models assume that when inventory is replenished, leftover inventory from the previous cycle is offered to customers alongside fresh units (see Baker and Urban, 1988). The analysis and implementation of such models is highly complex in cases of items whose demand is sensitive to freshness. Second, the assumption that inventory level is known to the customers is not necessarily accurate. For example, in many cases customers see only part of the inventory (e.g., in supermarkets), while additional stock is placed in a warehouse or other backroom storage area (Urban, 2002). Wu et al. (2006), You and Hsieh (2007), and Chang et al. (2010) propose models that assume an inventory-level-dependent demand rate and take inventory deterioration or perishability into account. The models proposed by You and Hsieh (2007) and Chang et al. (2010) also incorporate the effects of price on the demand rate.

Recent studies in inventory management of a single product with a deterministic demand rate have considered how demand rate is affected by both price and elapsed time, measured from the most recent replenishment. You (2005) and Tsao and Sheen (2008) studied a case with a known sales season, a linear price effect and an exponential time effect on the demand rate, an opportunity to adjust prices in fixed time-intervals, and no backlogging. While You (2005) assumed an additive influence of price and time on the demand rate, Tsao and Sheen (2008) assumed a multiplicative influence of these factors. Both Valliathal and Uthayakumar (2011) and Maihami and Kamalabadi (2012) assumed a multiplicative influence of price and time on the demand rate with partial backlogging. Maihami and Kamalabadi (2012) studied a general exponential time effect (even when inventory is exhausted) that could be either positive or negative (for deteriorating items), whereas Valliathal and Uthayakumar (2011) required only that the time effect be a non-negative continuous function. The two studies formulated a profit maximization problem with three decision variables: price, replenishment period and the time at which shortage starts, and suggested two-dimensional searches to locate the optimum. However, it seems that the papers do not fully prove that their respective searches converge to optimal solutions. Avinadav et al. (2013) also assumed a multiplicative effect of price and time on the demand rate but without backlogging. Their paper studied a case in which the price effect is negative-linear (similar to Maihami and Kamalabadi, 2012) and the time effect is negative-polynomial (similar to Avinadav and Arponen, 2009). They showed that the three-variable (order quantity, replenishment period and price) profit maximization problem can be reduced into a single-variable (replenishment period) problem, whose objective function is quasi-concave. Consequently, the optimal replenishment period can be obtained using an efficient line-search, and then the other decision variables can be calculated analytically.

3. Model formulation and general properties of the optimal policy

We formulate a model to obtain the optimal order quantity, replenishment period and selling price of perishable items when demand rate is affected by price and inventory age. It is assumed that the age of an item at delivery time is zero, so that the elapsed time measured from the most recent replenishment indicates the inventory age. We allow the inventory age effect on demand to be either negative or zero. In contrast to Valliathal and Uthayakumar (2011) and Maihami and Kamalabadi (2012), backlogging is not considered, as it is not common for most perishable items, which are at the focus of this work. First, when a retailer stocks multiple brands (e.g. milk of competitive dairy producers), shortage of a single brand followed by backlogging is less likely to occur (see, e.g., Krishna, 1992, p. 268), since the likelihood that consumers are willing to wait for replenishment is low. Second, perishable items are usually purchased to satisfy immediate needs of the consumer, so that shortages lead to loss of sales.

We generalize the work of Avinadav et al. (2013) as follows: First, the separability of the demand function into price and inventory age is expressed either by a multiplicative or by an additive model. Second, the effects of price and inventory age on the demand rate are not limited to specific functions, but to decreasing and non-increasing functions, respectively. For each model we show that the optimal solution satisfies the basic law in economics: marginal revenue equals marginal cost including the associated inventory holding costs. By solving the optimality equation, we reduce the profit-maximization problem into a single-variable problem (in which the variable is the replenishment period) that can be solved using a line-search. We also provide a sufficient condition for quasi-concavity of the single-variable profit function, which renders the efficient line-search