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Allocational flexibility in constrained supply chains

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ABSTRACT

We show that when a one-supplier/one-newsvendor supply chain is capacity-constrained, wholesale price contracts have some flexibility in allocating the channel-optimal profit. We analyze how this flexibility changes as we change the supply chain's capacity constraint and market demand. We also explore the allocation that is achieved in equilibrium in a newsvendor procurement game. Finally, we generalize our results to risk-sharing contracts and show that those contracts also gain additional flexibility in allocating the channel-optimal profit.

1. Introduction

There is a wealth of supply contracts available that coordinate a newsvendor's decision for unconstrained supplier–retailer channels: buy-back contracts, revenue-sharing contracts, etc. (Cachon, 2003). A contract *coordinates* the actions of a newsvendor *for* a supply channel if the contract causes the newsvendor to take actions when solving his *own* decision problem that are also optimal for the *channel*. Our paper starts from the fact that simpler contracts, namely linear wholesale price contracts, (which are thought to be unable to coordinate a newsvendor's decision for unconstrained channels) can, in fact, coordinate a newsvendor's procurement decision for resource-constrained channels. This is relevant for supply channels in which capacity of some resource is limited. For example, shelf space at retail stores, warehouse space, procurement budgets, time available for manufacturing, raw materials, etc. We show that in addition to having this coordination capability, in constrained supply channels, wholesale price contracts also have flexibility in allocating profit while maintaining coordination.

However, this extra gain in *allocational flexibility* is not limited to wholesale price contracts. We also show that when the channel is constrained, buyback contracts also gain some additional allocational flexibility. In particular, we show that buyback contracts gain a feature that they do not have in the unconstrained setting: the flexibility in allocating channel optimal profit, for any fixed level of risk.

Wholesale price contracts are commonplace since they are straightforward and easy to implement. While risk-sharing contracts such as revenue-sharing agreements can coordinate a retailer's decision in a newsvendor setting, Cachon and Lariviere (2005) note that these alternative contracts impose a heavier administrative burden. For example, these alternative contracts may require an investment in information technology or a higher level of trust between the trading partners due to the additional processes involved. In this paper, we show that the flexibility gained by wholesale price contracts in allocating the channel-optimal profit makes these simpler contracts more efficient and appropriate for a wider variety of supply channels than previously known.

Furthermore, after analyzing the allocational flexibility of wholesale price contracts, we analyze an equilibrium setting, where choosing the wholesale price is an initial stage of a game for the supplier. In the equilibrium setting we explore conditions for the game's equilibrium wholesale price to coordinate the newsvendor's procurement decision for the channel (i.e., necessary and sufficient conditions so that the game's equilibrium is included in the set of coordinating wholesale price contracts) and find the equilibrium profit allocation achieved.

The organization of this paper is as follows. In Section 2, we provide an overview of the supply contracts literature and in Section 3, we provide a stylized 1-supplier/1-retailer model. In Section 4, we show that wholesale price contracts have some flexibility in allocating the channel-optimal profit between the supplier and retailer (a flexibility that does not exist in the unconstrained setting). We also conduct some comparative statics and analyze how this flexibility changes as a function of capacity and market demand. Then in Section 5 we move on and consider risk-sharing contracts for the same supply chain model. We show that they still coordinate a capacity-constrained channel and,

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furthermore, there is even more flexibility in the choice of risk-sharing contracts (for coordinating the channel). In particular, for any given level of risk (represented by the buyback parameter of a buyback contract), there is now flexibility in allocating the channel profit (without sacrificing coordination), a flexibility that is not present in the unconstrained setting. Then, in Section 6 we analyze the equilibrium of a newsvendor procurement game in order to find and analyze the equilibrium profit allocation. Finally, we summarize our findings and provide managerial insights in Section 7.

2. Literature review

The supply contracts literature has been based on the observation, pointed out, for example, by Lariviere and Porteus (2001), that wholesale price contracts are simple but *do not* coordinate the retailer's order quantity decision for a supplier–retailer supply chain in a newsvendor setting and have no flexibility in allocating the channel-optimal profit. This observation has led to the study of an assortment of alternative contracts. For example, buy back contracts (Pasternack, 1985), quantity flexibility contracts (Tsay, 1999), and many others. Cachon (2003) provides an excellent survey of the many contracts and models that have been studied in the supply contracts literature. The mindset surrounding wholesale price contract's inability to channel-coordinate is true under appropriate assumptions—which the supply contracts literature has been implicitly assuming: that there are no capacity constraints (e.g., shelf space and budget).

Considering capacity constraints in a supply channel is not new to the supply contracts literature. However, most other papers in the literature consider choosing capacity as one stage of a game (before downstream demand is realized) that also involves a production decision after demand is finally realized (Cachon and Lariviere, 2001; Gerchak and Wang, 2004; Wang and Gerchak, 2003; Tomlin, 2003). Our paper, although complementary to this stream of literature, does not involve an endogenous capacity choice for any party but rather analyzes how an exogenous capacity constraint determines the set of wholesale prices that can coordinate the retailer's decision for the channel and the possible allocations of channel-optimal profit. Pasternack (2001) considers an exogenous budget constraint, but not for the purposes of studying coordination or allocational flexibility. Rather, he analyzes a retailer's optimal procurement decision when the retailer has two available strategies: buying on consignment and outright purchase.

Also our paper is not the first to reconsider wholesale price contracts and their benefits beyond simplicity. Cachon (2004) looks at how inventory risk is allocated according to wholesale price contracts and the resulting impact on supply chain efficiency. As far as we are aware, our paper is the first to consider the allocational flexibility of linear wholesale price contracts under a simple capacity-constrained production/procurement newsvendor model.

3. Model

A risk-neutral retailer r faces a newsvendor problem in ordering from a risk-neutral supplier for a single good: there is a single

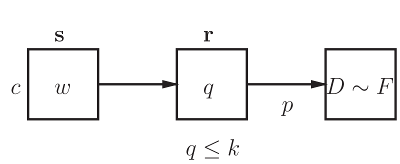

Fig. 1. "Single supplier & single capacity constrained retailer" model.

sales season, the retailer decides on an order quantity q and orders well in advance of the season, the entire order arrives before the start of the season, and finally demand is realized, resulting in sales for the retailer (without an opportunity for replenishment). Without loss of generality, we assume that units remaining at the end of the season have no salvage value and that there is no cost for stocking out.

The model's parameters are summarized in Fig. 1 with the arrows denoting the direction of product flow. In particular, the supplier has a fixed marginal cost of c per unit supplied and charges the retailer a wholesale price $w \geq c$ per unit ordered. The retailer's price p per unit to the market is fixed, and we assume that $p > w$. For that price, the demand D is random with probability density function (p.d.f.) f and cumulative distribution function (c.d.f.) F. We also define $\overline{F}(x) \overset{\text{def}}{=} 1 - F(x) = P(D > x)$. We say that a c.d.f. F has the *IGFR property* (increasing generalized failure rate), if $g(x) \overset{\text{def}}{=} x \cdot f(x)/\overline{F}(x)$ is weakly increasing on the set of all x for which $\overline{F}(x) > 0$ (Lariviere and Porteus, 2001). Most distributions used in practice (such as the Normal, the Uniform, the Gamma, and the Weibull distribution) have the IGFR property.

We assume that the retailer's capacity is constrained by some $k > 0$; for example, the retailer can only hold k units of inventory, or accept a shipment not larger than k. For a different interpretation, k could represent a constraint on the capacity of the channel or a budget constraint.

Assumption 1. The probability density function (p.d.f.) f for the demand D has support $[0, l]$, with $l > k$, on which it is positive and continuous.

As a consequence, $\overline{F}(0) = 1$ and $\overline{F}$ is continuously differentiable, strictly decreasing, and invertible on $(0, l)$. There is no additional restriction on the value of l. This is not a restrictive assumption and is made for technical reasons as shown in our proofs.

3.1. Retailer's problem

Faced with uncertain sales $S(q) \overset{\text{def}}{=} \min\{q, D\}$ (when ordering q units) and a wholesale price w (from the supplier), the retailer decides on a quantity to order from the supplier in order to maximize expected profit $\pi_r(q) \overset{\text{def}}{=} E[pS(q)] - wq$ while satisfying the capacity constraint k. Namely, it solves the following concave optimization problem with linear constraints in the decision variable, q:

RETAILER(k,w)

$$\begin{aligned} \text{maximize} \quad & pE[S(q)] - wq \\ \text{subject to} \quad & k - q \geq 0 \\ & q \geq 0. \end{aligned} \tag{1}$$

Because of our assumptions on the c.d.f. F, it can be shown that *RETAILER(k,w)* has a unique solution which we denote by $q^r(w)$.

3.2. Channel's problem

Denote the channel's expected profit by $\pi_s(q) \overset{\text{def}}{=} E[pS(q) - cq]$. Under capacity constraint k, the optimal order quantity q^s for the system/channel is the solution to concave optimization problem (2), *CHANNEL(k)*. Note that *CHANNEL(k)* has identical linear constraints but a slightly altered objective function when compared to *RETAILER (k,w)*:

CHANNEL(k)

$$\begin{aligned} \text{maximize} \quad & pE[S(q)] - cq \\ \text{subject to} \quad & k - q \geq 0 \\ & q \geq 0 \end{aligned} \tag{2}$$

Again because of our assumptions on the c.d.f. F it can be shown that *CHANNEL(k)* also has a unique solution which we denote by q^s.