Int. J. Production Economics 153 (2014) 238–242

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Batch scheduling with a rate-modifying maintenance activity to minimize total flowtime

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ARTICLE INFO

Article history:
Received 6 November 2012
Accepted 5 March 2014
Available online 15 March 2014

Keywords:
Batch scheduling
Single machine
Rate-modifying maintenance activity
Flowtime
Setup time

ABSTRACT

We study a single machine batch scheduling problem with unit time jobs and an optional maintenance activity. The maintenance activity is assumed to be rate modifying, i.e. the processing times of the jobs processed after the maintenance are reduced. The objective function is minimum total flowtime. We focus first on the relaxed version of the problem, where batch sizes are not forced to be integers. For a given number of jobs, setup time, duration of the maintenance activity, and a rate-modifying factor, we show that the optimal solution has a unique property: the batch sizes of the jobs scheduled prior to the maintenance, and after it, form two decreasing arithmetic sequences. Based on this property, we introduce an optimal algorithm which is polynomial in the number of jobs. We propose a simple rounding procedure that guarantees an integer solution. Our numerical tests indicate that this procedure leads to very close-to-optimal schedules.

1. Introduction

In the last two decades, many researchers of scheduling models consider the very realistic option of performing a maintenance activity. In most applications, during the maintenance time, the machine is unavailable, i.e. the production is stopped. We refer the reader to the survey paper "Machine scheduling with availability constraints" (Lee, 2004). This survey provides various settings of unavailability: scheduling on a single-machine, parallel machines, and shops, assuming different objective functions, and considering resumable or non-resumable processing. Lee and Leon (2001) focused on scheduling a maintenance activity which is *rate-modifying*, i.e., they assumed, as in many real-life systems, that the maintenance improves the performance of the machine, and the processing times of the jobs scheduled after the maintenance are reduced. Several extensions of this model have been studied; see e.g., Mosheiov and Oron (2006), Gordon and Tarasevich (2009), Mosheiov and Sarig (2009), Zhao et al. (2009), Ji and Cheng (2010), Lodree and Geiger (2010), Wang and Wang (2010), Mor and Mosheiov (2012,b), and Zhao and Tang (2012), among others.

Another popular topic in scheduling research is that of *batch scheduling*. In this class of problems, jobs are grouped and scheduled in batches, and in most cases a setup time is incurred when starting a new batch. The job allocation to batches is the main decision to be made, and the difficulty is due to the trade-off between minimizing the total setup time (achieved by having a small number of a relatively large-size batches), and minimizing the job waiting time within a batch (achieved by increasing the number of small batches). Some relevant studies are Cheng et al. (1996), Ghosh and Gupta (1997), Guoqing and Cheng (2000), Lin and Jeng (2004), Yuan et al. (2007), and Lin and Liao (2012). We refer the reader also to the survey paper of Allahverdi et al. (2008), which contains a long list of references dealing with batch scheduling on a large number of machine settings, and various objective functions. One of the classical batch scheduling models was introduced by Santos and Magazine (1985). They considered a single machine, identical jobs, identical (batch-independent) setup times, and the objective of minimum flowtime. This problem and many extensions of it have been studied extensively, e.g. by Dobson et al. (1989), Naddef and Santos (1988), Coffman et al. (1989), Shallcross (1992), Mosheiov et al. (2005), Quadt and Kuhn (2007) and Mor and Mosheiov (2011, 2012a), among others.

In this paper we combine the two areas of "scheduling a maintenance activity" and "batch scheduling". For a specific application, we extend the setting given in Cheng et al. (2008), who consider telecommunication services. Here, the objective is that of satisfying the service requirements of a content provider, using a commercial satellite to transfer voice, image and text files for his clients. Each type of data is partitioned into identical sized packets, and these are grouped into batches, and transmitted via a

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http://dx.doi.org/10.1016/j.ijpe.2014.03.004
0925-5273/

transponder in a time multiplexed technique. The transponder needs software updates from time to time, during which the content transmission is stopped. These time intervals are, in fact, classical maintenance activities. One relevant objective function in this context (also considered by Cheng et al. (2008)) is minimum flowtime, which is the objective considered in this paper. The specific model studied in this paper combines the rate-modifying model of Lee and Leon (2001), and the batch scheduling model of identical jobs of Santos and Magazine (1985). We show that the combined problem can be formulated as a quadratic programming problem. The optimal solution of the "relaxed" version of the problem, allowing non-integer batch sizes, consists of two (non-identical) decreasing arithmetic sequences of batch sizes: before and after the maintenance activity. Based on this unique property, we propose a solution procedure requiring a total computational effort of $O(n^2)$ (where n is the number of jobs). We also introduce a simple rounding procedure which leads to integer batch sizes, and results in extremely small optimality gaps. [It should be emphasized that despite the efficiency of our proposed algorithm, it is *not polynomial* in the input size, which contains only four parameters: the number of jobs, the setup time, the maintenance duration and the rate-modifying factor.]

This paper is organized as follows. In Section 2, we formulate the problem. In Section 3, we introduce an optimal solution for the relaxed version, and Section 4 contains the rounding procedure. Sections 5 and 6 present numerical examples and numerical tests, respectively.

2. Formulation

Formally, n jobs need to be processed on a single machine. All the jobs are available at time zero, and preemption is not allowed. We assume unit processing time for all the jobs. Jobs may be processed in batches. A setup time is required when starting a new batch. We assume integer setup time denoted by s. For a given job allocation to batches, let m denote the number of batches. Let B_j, $j=1, \ldots, m$, denote batch j, and $n_j=|B_j|>0$, $j=1, \ldots, m$, denote the number of jobs assigned to batch B_j. Clearly, $n=\sum_{j=1}^{m} n_j$. An optional Rate-Modifying maintenance Activity (RMA), with fixed length denoted by T, may be performed. The rate modifying factor is $\alpha \in (0,1)$, i.e., the processing time of a job (which is 1 if scheduled prior to the maintenance) becomes α if it is scheduled after the maintenance. Thus, if batch B_j is scheduled after the RMA, its processing time is αn_j, $j=1,\ldots,m$.

For a given allocation of jobs to batches, let C_j denote the completion time of batch B_j, $j=1,\ldots,m$. We assume *batch availability*, i.e., the completion time of a job is identical to the completion time of the batch to which it is assigned. The objective function is total flowtime, and the contribution of batch B_j to the flowtime is given by n_jC_j. Thus, the objective function is given by $\sum C=\sum_{j=1}^{m} n_jC_j$. Using the conventional notation, the problem studied in this paper is $1/batch, p=1,\ s, RMA/\sum C_j$.

3. An optimal solution for the relaxed version

In this section we focus on solving the relaxed version of the problem, where the batch sizes are not necessarily integers. Thus, the (optimal) solution obtained here is a *lower bound* on the true optimal flowtime for the original problem, where batch sizes are forced to be integers. (As mentioned, a simple rounding procedure is presented later.)

It is trivial to show that an optimal schedule exists with no idle time between consecutive jobs. Since RMA is optional, one candidate for optimality is a schedule without performing RMA. Another special candidate is when RMA is performed at time zero and affects all the batches. For all other candidates, we prove the following property:

Property 1. *An optimal schedule exists such that RMA is performed at the completion time of batch B_j, $j=1,\ldots,m-1$.*

Proof. Assume that RMA is performed after the completion time of $N>0$ jobs in batch B_k. (There are n_k-N jobs in this batch scheduled after RMA.) When the job scheduled immediately prior to RMA is rescheduled to be processed immediately after it, its processing time is clearly reduced by $1-\alpha$. It follows that the completion time of the entire batch is reduced by $1-\alpha$. Repeating this procedure leads to the conclusion that scheduling RMA prior to the first job of the batch is always preferred. □

Following Property 1, let k denote the index of the first batch assigned after RMA. For a given k, $k=1,\ldots,m,m+1$, we denote by $f(k)$ the resulting flowtime. [Note the above mentioned two special cases: $f(1)$ denotes the case that RMA is performed at time zero, and $f(m+1)$ denotes the case that RMA is not performed at all.] For $k=1,\ldots,m+1$, the objective function is given by the following:

$$
\begin{aligned}
f(k) &= \Big[(s+n_1)n_1+(2s+n_1+n_2)n_2\cdots+\Big((k-1)s+\textstyle\sum_{j=1}^{k-1}n_j\Big)n_{k-1} \\
&\quad+\Big(ks+\textstyle\sum_{j=1}^{k-1}n_j+T+\alpha n_k\Big)nk+\cdots \\
&\quad+\Big(ms+\textstyle\sum_{j=1}^{k-1}n_j+T+\alpha\sum_{j=k}^{m}n_j\Big)n_m\Big] \\
&= \Big[\textstyle\sum_{j=1}^{k-1}\Big(\sum_{i=1}^{j}n_i\Big)n_j+\Big(\sum_{i=1}^{k-1}n_i\Big)\Big(\sum_{j=k}^{m}n_j\Big) \\
&\quad+\alpha\textstyle\sum_{j=k}^{m}\Big(\sum_{i=k}^{j}n_i\Big)n_j+s\sum_{j=1}^{k-1}jn_j+s\sum_{j=k}^{m}jn_j+T\sum_{j=k}^{m}n_j\Big] \\
&= \Big[\textstyle\sum_{j=1}^{m}\Big(\sum_{i=1}^{j}n_i\Big)n_j-\sum_{j=k}^{m}\Big(\sum_{i=k}^{j}n_i\Big)n_j \\
&\quad+\alpha\textstyle\sum_{j=k}^{m}\Big(\sum_{i=k}^{j}n_i\Big)n_j+s\sum_{j=1}^{m}jn_j+T\sum_{j=k}^{m}n_j\Big] \\
&= \Big[\textstyle\sum_{j=1}^{m}\Big(\sum_{i=1}^{j}n_i\Big)n_j-(1-\alpha)\sum_{j=k}^{m}\Big(\sum_{i=k}^{j}n_i\Big)n_j \\
&\quad+s\textstyle\sum_{j=1}^{m}jn_j+T\sum_{j=k}^{m}n_j\Big].
\end{aligned}
$$

We use the equality $\sum_{j=1}^{n}\Big(\sum_{i=1}^{j}\alpha_i\Big)\alpha_j=\frac{1}{2}\sum_{i=1}^{n}\alpha_i^2+\frac{1}{2}(\sum_{i=1}^{n}\alpha_i)^2$, to rewrite the first two terms as follows:

$$\sum_{j=1}^{m}\Big(\sum_{i=1}^{j}n_i\Big)n_j=\frac{1}{2}\sum_{j=1}^{m}n_j^2+\frac{1}{2}\Big(\sum_{j=1}^{m}n_j\Big)^2=\frac{1}{2}\sum_{j=1}^{m}n_j^2+\frac{1}{2}n^2,$$

$$(1-\alpha)\sum_{j=k}^{m}\Big(\sum_{i=k}^{j}n_i\Big)n_j=\frac{1}{2}(1-\alpha)\sum_{j=k}^{m}n_j^2+\frac{1}{2}(1-\alpha)\Big(\sum_{j=k}^{m}n_j\Big)^2.$$

Thus, the objective function expression is easily converted to the following:

$$
\begin{aligned}
f(k) &= \left[\frac{1}{2}\sum_{j=1}^{m}n_j^2+\frac{1}{2}n^2-\frac{1}{2}(1-\alpha)\sum_{j=k}^{m}n_j^2-\frac{1}{2}(1-\alpha)\left(\sum_{j=k}^{m}n_j\right)^2\right. \\
&\quad\left.+s\sum_{j=1}^{m}jn_j+T\sum_{j=k}^{m}n_j\right].
\end{aligned}
\tag{1}
$$

Thus, the formal problem is minimizing (1), subject to the constraint that the sum of all (non-negative) batch sizes is n

Min $f(k)$

(i) $\sum_{j=1}^{m}n_j=n$;
(ii) $n_j\geq 0$, $j=1,..,m$.

The objective function is a quadratic function of the batch sizes, and the constraints are linear. Since we assume strictly positive