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Coordinated scheduling on parallel machines with batch delivery

Long Wan [a], An Zhang [b,*,1]

[a] School of Information Technology, Jiangxi University of Finance and Economics, PR China
[b] Institute of Operational Research and Cybernetics, Hangzhou Dianzi University, Hangzhou 310018, PR China

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ABSTRACT

This paper considers coordinated scheduling on parallel identical machines with batch delivery. Jobs are first processed on m parallel and identical machines in the manufacturing facility and then delivered to the customer in batches. There are v identical transporters that can carry up to c jobs in one shipment. The objective is to minimize the sum of job arrival times. We show that the problem is NP-hard in the strong sense if m is part of the input. Besides, we propose the first approximation algorithm for the problem and prove that the worst case ratio of the algorithm is $2-1/m$ for any m.

1. Introduction

The classical scheduling problems usually assume that either there is an infinite number of transporters for delivering jobs or the transportation between the facility and the customers can be done instantaneously. As a result, the machine scheduling and the job delivery are separately and sequentially considered without effective coordination between the two. However, in most manufacturing and distribution systems, these assumptions seem too ideal and making two decisions separately without coordination will not necessarily yield a global optimal solution. Coordination between machine scheduling and job delivery becomes more practical.

In the last decade, the coordination of scheduling and transportation has become one of the most important topics in production and operations management research. Lee and Chen (2001) consider two types of transportation. The first type is intermediate transportation in a flow shop where jobs are transported from one machine to another for further processing. The second type is the transportation necessary to deliver finished jobs to the customer. An extended model in which each job occupies a different amount of storage space in the transporter is introduced by Chang and Lee (2004). For both single machine and two-parallel machine scheduling problems, approximation algorithms are presented. He et al. (2006), Zhong et al. (2007) and Su et al. (2009) revisit the problem of Chang and Lee (2004) and get improved algorithms.

There are also researchers considering the delivery cost of batches. Wang and Cheng (2000) study a parallel machine scheduling with batch delivery, in which they show that the problem to minimize the sum of job arrival times and delivery cost is NP-hard in the ordinary sense even when only two machines are available and NP-hard in the strong sense when the number of machines is part of the input. Hall and Potts (2003) consider a variety of single machine scheduling problems with the objective of minimizing the overall scheduling and delivery cost. Complexity results and dynamical programming algorithms are given. Note in their model, it is assumed that there are always transporters available for delivering. In Hall and Potts (2005), Hall and Potts analyze the complexity for many different objectives on single or parallel machine scheduling with batch delivery. For other researches, we refer to Chen (1996), Yuan (1996), Ji et al. (2007), Gong and Tang (2008), and Selvarajah and Zhang (2014).

In this paper, we investigate the problem of coordinated scheduling on parallel identical machines with batch delivery. We are given a set of n jobs $J_1, J_2, \ldots, J_n$ in the manufacturing facility which should be non-preemptively processed by m parallel and identical machines. Finished jobs must be delivered to the warehouse or the customer in batches, where a batch is defined as the jobs delivered in one shipment. There are v identical transporters located at the manufacturing facility initially. Each transporter can carry up to c jobs in one shipment. The transportation time from the manufacturing facility to the customer is t_1 and that from the customer to the manufacturing facility is t_2. We suppose that the transportation time is independent of the jobs being delivered. Let p_j be the processing time of job J_j and D_j be its arrival time, i.e, the time when J_j arrives at the customer. We study the problem of minimizing the sum of job arrival times. According to Lee and Chen (2001), this problem is denoted by $Pm \rightarrow D|v \geq 1, c \geq 1|\sum D_j$. If only one transporter is available, i.e, $v=1$, Lee and Chen (2001) have shown the problem when $m=2$ is NP-hard in the ordinary sense, we prove that it is strongly

* Corresponding author. Tel.: +86 571 86878594; fax: 86 571 87953832.
E-mail addresses: cocu3328@163.com (L. Wan), anzhang@hdu.edu.cn (A. Zhang).
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NP-hard if the number of machines is part of the input. In addition, we propose the first approximation algorithm for the general problem $Pm \to D|v \geq 1, c \geq 1|\sum D_j$, which has a worst case ratio of $2-1/m$ for any m.

The paper is organized as follows. In Section 2 we give some preliminaries and useful lemmas. Section 3 studies the complexity of the problem. In Section 4, we propose the SPT-DP algorithm to solve the problem. Finally, some conclusions are made in Section 5.

2. Preliminaries

Although the problem considered in this paper is different from that in Hall and Potts (2005), some properties can be found in the same way.

Lemma 2.1 (*Hall and Potts, 2005*). *For problem $Pm \to D|v \geq 1$, $c \geq 1|\sum D_j$, there exists an optimal solution in which: (1) jobs are scheduled without idle times between each other; (2) all deliveries are made either at job completion times or immediately the transporter becomes available.*

Lemma 2.2 (*Lee and Chen, 2001*). *For problem $Pm \to D|v \geq 1$, $c \geq 1|\sum D_j$, there exists an optimal solution in which: (1) each delivery batch contains consecutively completed jobs; (2) earlier completed jobs are delivered no later than those completed later.*

Since both scheduling and delivery are involved in our problem, we may make two types of decisions in an algorithm: one about the scheduling and the other about the delivery policy. For the scheduling part, a feasible schedule of jobs will be given to assign each job to a machine without preemption. For the delivery part, we make a batching and delivery policy for jobs, which means, by Lemmas 2.1 and 2.2, that we should decide the number of jobs of each batch.

Let π and π^* be a feasible solution and an optimal solution respectively, we will use the following notations and definitions frequently in the reminder of this paper.

k^π (k^*)	The total number of batches that $\pi(\pi^*)$ used.
B_i^π (B_i^*)	The i-th delivered batch in $\pi(\pi^*)$.
n_i^π (n_i^*)	The number of jobs delivered up to the i-th batch, clearly $n_{k^\pi}^\pi = n_{k^*}^* = n$.
C_j^π (C_j^*)	The completion time of job J_j in the manufacturing facility.
$C_{[j]}^\pi$ $(C_{[j]}^*)$	The completion time of the j-th completed job in the manufacturing facility.
D_j^π (D_j^*)	The arrival time of J_j, i.e., the time when J_j arrives at the customer.
$D^\pi(i)$ $(D^*(i))$	The arrival time of jobs in the i-th batch.

Clearly if we let $n_0^\pi = n_0^* = 0$, then the total number of jobs in the i-th batch is exactly $n_i^\pi - n_{i-1}^\pi$ $(n_i^* - n_{i-1}^*)$. Accordingly, the objective values of π and π^* can be calculated, i.e., $\sum_{j=1}^n D_j^\pi = \sum_{i=1}^{k^\pi}(n_i^\pi - n_{i-1}^\pi)D^\pi(i)$ and $\sum_{j=1}^n D_j^* = \sum_{i=1}^{k^*}(n_i^* - n_{i-1}^*)\ D^*(i)$. Since it is clear that $D_j^\pi \geq C_j^\pi + t_1$ $(D_j^* \geq C_j^* + t_1)$ and $D^\pi(i) \geq\ C_{[n_i^\pi]}^\pi + t_1$, the following lemma can be obtained easily.

Lemma 2.3. *For any solution π to the problem $Pm \to D|v \geq 1, c \geq 1|\sum D_j$,*

(1) $\sum_{j=1}^n D_j^\pi = \sum_{i=1}^{k^\pi}(n_i^\pi - n_{i-1}^\pi)D^\pi(i) \geq \max\{\sum_{j=1}^n C_j^\pi + nt_1, \sum_{i=1}^{k^\pi}(n_i^\pi - n_{i-1}^\pi)[C_{[n_i^\pi]}^\pi + t_1]\}$;

(2) *If $v=1$, then $D^\pi(i) \geq D^\pi(i-1) + (t_1+t_2) \geq D^\pi(1) + (i-1)(t_1+t_2)$ and hence $\sum_{j=1}^n D_j^\pi = \sum_{i=1}^{k^\pi}(n_i^\pi - n_{i-1}^\pi)D^\pi(i) \geq \sum_{i=1}^{k^\pi}(n_i^\pi - n_{i-1}^\pi)$ $[D^\pi(1) + (i-1)\ (t_1+t_2)]$, where the equality holds only when batches are delivered consecutively, i.e., each batch can be delivered immediately after it gets ready.*

Usually, the quality of an approximation algorithm (denoted by H) is measured by the *worst-case ratio* of the algorithm, which is defined as the smallest number ρ such that $F^H \leq \rho F^*$ for all instances, where F^H and F^* denote the objectives of solution produced by H and the optimal algorithm, respectively.

3. Complexity of problem $P_m \to D|v=1, c \geq 1|\sum D_j$

In this section, we study the complexity of problem $P_m \to D|v=1, c \geq 1|\sum D_j$. The main result is as follows.

Theorem 3.1. *The problem $P_m \to D|v=1, c \geq 1|\sum D_j$ is NP-hard in the strong sense for any $c \geq 1$ even when $t_1 = t_2$, if the number of machines is part of the input.*

We prove this by a reduction from the problem 3PP, which is known to be strongly NP-complete.

3-partition problem (*3PP in short*): Given $3u$ items, $A = \{1, 2, \ldots, 3u\}$, each item $j \in A$ has a positive integer size a_j satisfying $B/4 < a_j < B/2$ and $\sum_{j=1}^{3u} a_j = uB$ for some integer B and u, the problem asks whether there exist u disjoint subsets $A_1, A_2, \ldots, A_u$ of A such that each subset contains exactly three items and its total size is equal to B.

Given an instance of 3PP, we construct the following instance of the scheduling problem.
Number of machines $m=u$.
Number of jobs $n = 4u$.
Capacity of the transporter c, where $1 \leq c \leq n$.
Processing time of jobs $p_i = (i-1)(2M + B/3u)$ for $1 \leq i \leq u$ and $p_i = 2uM + a_{i-u}$ for $u+1 \leq i \leq 4u$, where $M > (8/3)uB$ is a constant.
One-way delivery time $t_1 = t_2 = M + B/6u$. Note we have $2M + B/3u = t_1 + t_2$.

Let $y = 8u^2(2M + B/3u)$ be a threshold for the sum of job arrival times.

Lemma 3.2. *If there is a solution to the 3PP instance, then there is a solution to the scheduling instance with the total arrival time of jobs no more than y.*

Proof. Let $A_i = \{i_1, i_2, i_3\}$ with $1 \leq i_l \leq 3u, l = 1, 2, 3$ be the solution to the 3PP instance, $i = 1, 2, \ldots, u$. Then it follows that $a_{i_1} + a_{i_2} + a_{i_3} = B$ for all $1 \leq i \leq u$. W.l.o.g., let $a_{i_1} \leq a_{i_2} \leq a_{i_3}$, then for any $l = 1, 2, 3$,

$$\sum_{k=1}^{l} a_{i_k} \leq \frac{lB}{3}. \tag{1}$$

Now we construct the following solution π to the scheduling instance (see Fig. 1). In the scheduling part, machine M_i processes exactly four jobs ordered by $J_i, J_{u+i_1}, J_{u+i_2}$ and J_{u+i_3}. So the completion time of each job in the facility can be calculated accordingly

$$C_i^\pi = p_i = (i-1)\left(2\,M + \frac{B}{3u}\right), \tag{2}$$

$$C_{u+i_l}^\pi = p_i + \sum_{k=1}^{l} p_{u+i_k} = (i-1)\left(2M + \frac{B}{3u}\right) + \sum_{k=1}^{l}(2uM + a_{i_k}) \leq (lu+i-1)\left(2M + \frac{B}{3u}\right) \tag{3}$$

for $1 \leq i \leq u$ and $l = 1, 2, 3$, where the last inequality is due to (1).