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A note on: "The effect of optimal advertising on the distribution-free newsboy problem"

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ABSTRACT

Recently, Lee and Hsu (2011) study the advertising effect on the distribution-free newsboy problem. In one of their results, they show that the optimal expenditure on advertising, the optimal order quantity and the optimal lower bound on the expected profit increase with the advertising effect parameters. Although this result holds in general, it requires additional assumptions. In this note we provide the necessary and sufficient conditions under which the statements of Lee and Hsu (2011) hold. These conditions are shown to be violated if the advertising expenditure is too low or there is a very small difference between the optimal profit and the profit without advertising.

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1. Introduction

The effect of advertising on the single period inventory problem, or the newsboy problem, has been first addressed by Khouja and Robbins (2003). They assume that the mean demand is increasing and concave in advertising expenditure and study three cases of demand variation as a function of advertising expenditure: (i) demand has constant variance, (ii) demand has a constant coefficient of variation, and (iii) demand has an increasing coefficient of variation. Lee and Hsu (2011) study the effect of advertising on the distribution-free newsboy problem. Using the bounds on the expected profit of the newsvendor problem given by Alfares and Elmorra (2005), they provide closed form solutions for the optimal order quantities and the optimal advertising policy for the three cases given in Khouja and Robbins (2003). They also provide sensitivity analysis of the optimal decisions to the parameters. One of their results is the following: the optimal advertising policy, the optimal order quantity and the optimal lower bound on the expected profit increase with the advertising effect parameters. In this study, we show that this result of Lee and Hsu (2011) requires additional assumptions on the cost parameters and advertising parameters. We also show that these assumptions are violated if the advertising expenditure is too low, or if the difference between the optimal profit and the profit without advertising is too small.

2. The analysis

We use the same notation in Lee and Hsu (2011):

$c>0$	unit cost
p	unit selling price, $p=(1+m)c>c$ with m being the markup rate
s	unit salvage value, $s=(1-d)c<c$ with d being the discount rate
$l=kc$	unit shortage penalty cost, with k being the shortage penalty rate
D	random demand with mean μ and variance σ^2
μ_0	expected demand without advertising (the original market size)
σ_0	standard deviation of the demand without advertising, $\sigma_0<\mu_0$
Q	order quantity
B	expenditure on advertising

The authors consider a distribution-free newsvendor problem with D being the random demand with mean μ, standard deviation σ and distribution G with $\mu>\sigma$. In each period, the decision maker needs to decide the expenditure on advertising B and the order quantity Q to maximize the expected profit against the worst possible distribution of demand.

2.1. Constant variance case (CVC)

In this case, μ is assumed to be a concave increasing function of B, i.e., $\mu=\mu_0+\mu_0\omega B^\alpha$ where ω and α are empirically determined positive constants that represent the effectiveness of advertising and $0<\alpha<1$ and $0\leq\omega$. Let $\pi(Q,B)$ denote the expected profit. The decision-maker's problem is to maximize $\pi(Q,B)$:

$$\max_{Q,B\geq 0}\pi(Q,B) \quad pE[\min(Q,D)]+sE[(Q-D)^+] - lE[(D-Q)^+]-cQ-B$$

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s.t. $\mu=\mu_0+\mu_0\omega B^\alpha$
$\sigma=\sigma_0$

Let B_1^*, Q_1^* and π_1^* denote the optimal advertising expenditure, the optimal order quantity and the optimal lower bound on the expected profit in the CVC respectively. The values of B_1^*, Q_1^* and π_1^* and their first order derivatives are given by the following (Lee and Hsu, 2011):

$$B_1^*=(K_1\alpha)^{1/(1-\alpha)} \tag{1}$$

$$Q_1^*=\mu_0+\mu_0\omega B_1^*+\frac{\sigma_0}{2}\left\{\left[\frac{k+m}{d}\right]^{1/2}-\left[\frac{d}{k+m}\right]^{1/2}\right\}$$

$$\pi_1^*=cm(\mu_0+\mu_0\omega(B_1^*)^\alpha)-c\sigma_0(kd+md)^{1/2}-B_1^*.$$

$$\frac{dB_1^*}{d\alpha}=B_1^*((1-\alpha)^{-2}\log(K_1\alpha)+(\alpha(1-\alpha))^{-1}) \tag{2}$$

$$\frac{dQ_1^*}{d\alpha}=\mu_0\omega B_1^{*\alpha}\log B_1^*+\mu_0\omega\alpha B_1^{*\alpha-1}\frac{dB_1^*}{d\alpha} \tag{3}$$

$$\frac{d\pi_1^*}{d\alpha}=K_1B_1^{*\alpha}\log B_1^* \tag{4}$$

where $K_1=cm\mu_0\omega$. Note that K_1 is a constant which is independent of α. The authors claim that the quantities in (2)–(4) are non-negative, i.e., $dB_1^*/d\alpha\geq 0$, $dQ_1^*/d\alpha\geq 0$ and $d\pi_1^*/d\alpha\geq 0$, hence the optimal expenditure on advertising, the optimal order quantity and the optimal lower bound increase with α. However, this result is not correct. Now we give the following proposition which provides the necessary and sufficient conditions for the non-negativity of the quantities in (2)–(4).

Proposition 1.

(i) B_1^* *increases with* α *if and only if*:

$$K_1\alpha e^{(1-\alpha)/\alpha}\geq 1. \tag{5}$$

(ii) Q_1^* *increases with* α *if and only if*:

$$K_1\alpha e^{1-\alpha}\geq 1. \tag{6}$$

(iii) π_1^* *increases with* α *if and only if*:

$$K_1\alpha\geq 1. \tag{7}$$

(iv) (7) $\Rightarrow$ (6) $\Rightarrow$ (5)

Proof.

(i) The first derivative of B_1^* with respect to α can be written as

$$\frac{dB_1^*}{d\alpha}=\frac{B_1^*}{1-\alpha}\left(\log B_1^*+\frac{1}{\alpha}\right). \tag{8}$$

We have $B_1^*/(1-\alpha)\geq 0$ from (1) and by definition of the parameters. Hence we need $\log B_1^*+1/\alpha\geq 0$ in order to have a non-negative derivative. Substituting (1) and after some algebra, this condition can be re-written as $(K_1\alpha)^{1/(1-\alpha)}e^{1/\alpha}\geq 1$ which reduces to the condition given in (5).

(ii) The first derivative of Q_1^* with respect to α can be re-written as

$$\frac{dQ_1^*}{d\alpha}=\frac{\mu_0\omega}{1-\alpha}B_1^*(\log B_1^*+1). \tag{9}$$

Since $(\mu_0\omega/(1-\alpha))B_1^*\geq 0$ from the definition of parameters and (1), one needs to check the non-negativity of $\log B_1^*+1$. Using (1) and rearranging the term, it can be concluded that the expression in (9) is non-negative if $\log K_1\alpha e^{1-\alpha}\geq 0$ which reduces to (6).

(iii) The first derivative of π_1^* with respect to α can be re-written as

$$\frac{d\pi_1^*}{d\alpha}=\frac{B_1^*\log B_1^*}{\alpha} \tag{10}$$

Since $cm\mu_0\omega B_1^{*\alpha}\geq 0$ from (1) and the parameter definitions, one needs to check the non-negativity of $\log B_1^*$. Using (1), one can write $\log B_1^*=\log K_1\alpha/(1-\alpha)$. Then the expression in (10) is non-negative if $\log K_1\alpha\geq 0$ which reduces to the condition given in (7).

(iv) $1\leq e^{1-\alpha}\leq e^{(1-\alpha)/\alpha}$ since $0<\alpha<1$. Therefore $K_1\alpha\leq K_1\alpha e^{1-\alpha}\leq K_1\alpha e^{(1-\alpha)/\alpha}$. □

2.2. Constant coefficient of variance case (CCVC)

In this case, advertising increases both μ and σ in a proportional fashion, i.e., CV is constant. Both μ and σ are concave increasing functions of B: $\mu=\mu_0+\mu_0\omega B^\alpha$ and $\sigma=\sigma_0+\sigma_0\omega B^\alpha$. Let B_2^*, Q_2^* and π_2^* denote the optimal advertising expenditure, the optimal order quantity and the optimal lower bound on the expected profit in CCVC and let $K_2=c\omega(m\mu_0-\sigma_0(kd+md)^{1/2})$. The values and first order derivatives of B_2^*, Q_2^* and π_2^* are given as follows (Lee and Hsu, 2011):

$$B_2^*=(K_2\alpha)^{1/(1-\alpha)}$$

$$Q_2^*=\mu_0+\mu_0\omega B_2^*+\frac{\sigma_0+\sigma_0\omega B_2^*\alpha}{2}\left\{\left[\frac{k+m}{d}\right]^{1/2}-\left[\frac{d}{k+m}\right]^{1/2}\right\}$$

$$\pi_2^*=cm(\mu_0+\mu_0\omega(B_2^*)^\alpha)-c(\sigma_0+\sigma_0\omega(B_2^*)^\alpha)(kd+md)^{1/2}-(B_2^*)$$

$$\frac{dB_2^*}{d\alpha}=B_2^*((1-\alpha)^{-2}\log(K_2\alpha)+(\alpha(1-\alpha))^{-1}) \tag{11}$$

$$\frac{dQ_2^*}{d\alpha}=w\left(B_2^{*\alpha}\log B_2^*+\alpha B_2^*\frac{dB_2^*}{d\alpha}\right)\left(\mu_0+\frac{\sigma_0(m+k-d)}{2(kd+md)^{1/2}}\right) \tag{12}$$

$$\frac{d\pi_2^*}{d\alpha}=K_2B_2^{*\alpha}\log B_2^*, \tag{13}$$

where $K_2=c\omega(m\mu_0-\sigma_0(kd+md)^{1/2})$. Note that $K_2=K_1-c\omega\sigma_0(kd+md)^{1/2}$ and hence $K_1\geq K_2$. Similar to K_1, K_2 is a constant which is independent of α. The authors claim that the quantities in (11)–(13) are non-negative, i.e., $dB_2^*/d\alpha\geq 0$, $dQ_3^*/d\alpha\geq 0$ and $d\pi_2^*/d\alpha\geq 0$. However, as in the CVC case, this result is not correct. The following proposition provides the necessary and sufficient conditions for the quantities in (11)–(13) to be non-negative:

Proposition 2.

(i) B_2^* *increases with* α *if and only if*

$$K_2\alpha e^{(1-\alpha)/\alpha}\geq 1. \tag{14}$$

(ii) Q_2^* *increases with* α *if and only if*

$$K_2\alpha e^{1-\alpha}\geq 1. \tag{15}$$

(iii) π_2^* *increases with* α *if and only if*

$$K_2\alpha\geq 1. \tag{16}$$

(iv) (16) $\Rightarrow$ (15) $\Rightarrow$ (14).
(v) (14) $\Rightarrow$ (5), (15) $\Rightarrow$ (7), (16) $\Rightarrow$ (7).

Proof. (i) to (iv): After some algebra, the first order derivatives of B_2^*, Q_2^* and π_2^* can be re-written as follows:

$$\frac{dB_2^*}{d\alpha}=\frac{B_2^*}{1-\alpha}\left(\log B_2^*+\frac{1}{\alpha}\right)$$

$$\frac{dQ_2^*}{d\alpha}=w\left(\mu_0+\frac{\sigma_0(m+k-d)}{2(kd+md)^{1/2}}\right)B_2^*(\log B_2^*+1)\frac{d\pi_2^*}{d\alpha}=\frac{B_2^*\log B_2^*}{\alpha}$$

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