Int. J. Production Economics 146 (2013) 167–177

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Effects of higher order moments on the newsvendor problem

Qi Sun [a], Yucheng Dong [b], Weidong Xu [c,*]

[a] School of Business Administration, Contemporary Business and Trade Research Center, Zhejiang Gongshang University, Zhangzhou 310018, China
[b] Business School, Sichuan University, Chengdou 610065, China
[c] School of Management, Zhejiang University, Hangzhou 310058, China

ARTICLE INFO

Article history:
Received 17 June 2012
Accepted 14 June 2013
Available online 29 June 2013

Keywords:
Stochastic programming
Newsvendor problem
Conditional value-at-risk
Moment matching

ABSTRACT

In this paper, we propose a stochastic programming model for the well-known single-period newsvendor problem by adopting the conditional Value-at-Risk (CVaR) as the risk metric in the objective function. The demand uncertainty is modeled in terms of discrete scenarios that reflect the empirical distributions implied by market demand data. Our numerical results demonstrate that the higher order moments (skewness and kurtosis) of demand have obvious effects on the newsvendor problem, and the stochastic programming framework provides a flexible and effective decision support tool for the newsvendor problem.

1. Introduction

The single-period newsvendor problem is the foundation of many inventory control models. In the classical version, a newsvendor is often optimized by the expected profit improvement, without consideration of any of risk measures. However, in practice, there are many counter examples indicating that managers' order decisions are not always consistent with the classical newsvendor problem. One main reason leads to the decision bias occurred in the newsvendor problem is that the rational newsvendor may have preferences other than risk-neutral. Since then, according to this empirical phenomenon, many studies have been conducted to modify the classical newsvendor model. Markowitz's (1952) mean-variance as a representative methodology in the framework of return-risk trade-off analysis has been widely used for inventory control, e.g., Lau (1980), Berman and Schnabel (1986), Chen and Federgruen (2000), Choi et al. (2008), Choi and Chiu (2012), Wu et al. (2009), Egri and Váncza (2012), Jammernegg and Kischka (2013), and the references therein. Applying the mean-variance model to analyze the newsvendor problem is formulated as minimization of the profit variance for a given expected profit. However, one undesirable property of adopting variance as a risk measure is its symmetrical treatment of both upside and downside deviations from the mean as its risk measure, which cannot be justified theoretically. This is especially true for skewed distributions. Therefore, in spite of the importance of mean-variance analysis in operations management, it is believed that the choice of using variance as a risk measure lacks both theoretical and practical justifications. Furthermore, to obtain closed form solutions, some special demand distribution functions such as power distribution must be assumed, see Chen and Federgruen (2000), Wu et al. (2009), and Murray et al. (2012). The above facts limit the use of the mean-variance model to deal with realistic newsvendor problems. Wang and Webster (2009) used loss aversion to model managers' decision-making behavior in the single-period newsvendor problem. Value at Risk (VaR) is frequently used as a measure of downside risk. Özler et al. (2009), Gan et al. (2004), and Tapiero (2005) incorporated the VaR concept to a newsvendor problem with a downside risk constraint, formulated as a mean-VaR model. Unlike VaR, the conditional Value-at-Risk (CVaR) is a coherentrisk measure in the sense of Artzner et al. (1999). Therefore, recently Chahar and Taaffe (2009) presented a CVaR approach to solving the AON demand selection problem under risk averse conditions against incurring several losses across consecutive periods in the short run if using a profit maximization approach based on a newsvendor-type model. CVaR as a coherent down-side risk measure is also used by Zhou et al. (2008), Chen et al. (2009), and Gotoh and Takano (2007) to study the newsvendor problems.

Besides the risk preferences observed in the newsvendor model, Natarajan et al. (2008) also showed that "one phenomenon in production is a 'lumpy' or sporadic demand which is characterized by infrequent and large demand. This demand pattern has been observed in parts and supply inventory systems such as large compressors and textile machines". Under the classical newsvendor model, Natarajan et al. (2008) described the asymmetry by the semivariance, and the optimal bounds are solved through a

* Corresponding author. Tel.: +86 13456968608; fax: +86 571 88206402.
E-mail addresses: sunqisjtu@yahoo.cn (Q. Sun), ycdong@scu.edu.cn (Y. Dong), weidxu@zju.edu.cn (W. Xu).

0925-5273/$ - see front matter
http://dx.doi.org/10.1016/j.ijpe.2013.06.019

second-order cone programming problem. Therefore, if the demand distribution is known to have nonzero skewness and kurtosis, then it is important to consider stochastic models that can accurately describe these characteristics in decision making.

Our motivation is based on the fact that all the above papers studying newsvendor problems do not consider the effects of higher order moments (i.e., skewness and kurtosis). In this paper we develop a flexible single-period stochastic programming model for the newsvendor problem. The stochastic evolution of demand is modeled in terms of discrete scenarios reflecting the empirical distributions implied by market demand data. We applied a moment-matching method introduced by Høyland and Wallace (2001) and Høyland et al. (2003) to generate scenarios of demand so as to match the target moments. The method is also used by Topaloglou et al. (2008a,b), and Xu et al. (2011) to describe the dynamic evolution of asset prices in portfolio management and option pricing. We specified the target moments as the first four moments (mean, variance, skewness, and kurtosis) of the demand for a product. Stochastic programming can accommodate different objective functions to capture the decision maker's risk bearing preferences, e.g., variances, penalties on shortfalls, and utility functions. We employed the CVaR metric in the objective function in order to minimize the expected shortfall beyond the VaR of losses at the end of the horizon. The numerical analysis clearly shows that the higher moments (skewness and kurtosis) of demand have obvious effects on the efficient frontier of the expected profit and CVaR measure of risk and optimal order quantity.

This paper is organized as follows. Some well-known results of the classical newsvendor problems are introduced in Section 2. Stochastic programming model of newsvendor problem adopting CVaR as the risk measure is proposed in Section 3. In Section 4, a numerical study is performed to show the effects of higher moments on the efficient frontier and optimal order quantity. The concluding remarks are given in Section 5.

2. Risk-neutral newsvendor problems

In this section, we briefly summarize some basic results of the classical single-period newsvendor problem.

2.1. Notations

First of all, let us introduce notations used in the newsvendor problem as follows:

ξ: daily demand for a product;
q: selling price per unit of the product;
c: cost per unit of the product;
r: salvage value per unit of the product;
s: shortage penalty per unit of the product;
x: daily order quantity of the product;

We assume the following condition through the paper:

Assumption 2.1. $r < c < q, s \geq 0$.

2.2. Profit maximization

For fixed x, the daily profit gained from each product is a random variable defined by

$$P(x,\xi) = q\min(\xi,x) + r\max(x-\xi,0) - s\max(\xi-x,0) - cx, \tag{1}$$

where the third term in the right-hand side represents an artificial penalty for opportunity cost. Eq. (1) can be rewritten as

$$\begin{aligned} P(x,\xi) &= q[x-(x-\xi)^+] + r(x-\xi)^+ - s(\xi-x)^+ - cx \\ &= q[x-(x-\xi)^+] + r(x-\xi)^+ - s[(x-\xi)^+ - x + \xi] - cx \\ &= (q+s-c)x - (q+s-r)(x-\xi)^+ - s\xi \\ &= ax - b\omega - s\xi, \end{aligned} \tag{2}$$

where $a = q+s-c$, $b = q+s-r$, $\omega = (x-\xi)^+$, and $(\cdot)^+$ is defined as $(x-\xi)^+ = \max\{0, x-\xi\}$.

The classical newsvendor model is based upon risk-neutral so that a newsvendor will select an order quantity to maximize the expected profit

$$\begin{aligned} \max_x \quad & E[P(x,\xi)] \\ s.t. \quad & x \in \mathbb{X}, \end{aligned} \tag{3}$$

where $\mathbb{X}$ is a convex set representing some additional constraints for the feasible order quantity sets, e.g.,

$$\mathbb{X} = \{x \in \mathbb{R} | x \geq 0\}. \tag{4}$$

Constraint condition (4) corresponds to the case where no negative order quantity is allowed. The aim in Eq. (3) is confined to the optimization of the expected value of a given profit measure, and does not take into account the risk of earning less than a desired profit or losing more than an acceptable level due to the randomness of demand. In this paper, a mean-CVaR model has been proposed to extend the classical newsvendor model by adopting the CVaR of net losses as a risk measure. As a downside risk measure, CVaR exhibits some better properties than variance and VaR in measuring risk and can be easily dealt with stochastic programming for general distribution functions in newsvendor problems. Therefore, CVaR, also called the mean excess loss, or the tail VaR, becomes more and more popular in operations management today. It is worthwhile to investigate the optimal ordering for a risk averse newsvendor under the combination of CVaR and expected profit. As a result, we will adopt the mean-CVaR model to investigate the effects of higher order moments on the newsvendor problems in this paper.

3. Minimization of CVaR in the newsvendor problems

3.1. Conditional VaR

In our single-period stochastic programming model for the newsvendor problems, we will minimize the conditional Value-at-Risk (CVaR) of losses at the end of the planning horizon. For a planning horizon T, let $l(x,\xi)$ denote the loss of the newsvendor problem with decision variable $x \in R$ and random variable $\xi \in R$ representing the value of underlying risk factors at T. For simplicity, we assume that the random variable $\xi \in R$ has a probability density $f(\xi)$. For a given decision x, the probability of the loss not exceeding a threshold β is given by the distribution function

$$F(x,\beta) = \int_{l(x,\xi) \leq \beta} f(\xi)\, d\xi.$$

Given a confidence level $\alpha \times 100\%$, the VaR of a decision x is equivalently defined as

$$VaR(x,\alpha) = \min\{\beta \in R : F(x,\beta) \geq \alpha\}.$$

CVaR is a risk measure and it minimizes the expected value in the tail (beyond a specific percentile α) of portfolio losses at the end of the planning horizon. CVaR is defined as the expected value of loss that exceeds VaR(z,β). For continuous function, it is,

$$CVaR(x,\alpha) = \frac{1}{1-\alpha}\int_{l(x,\xi) \geq VaR(x,\alpha)} l(x,\xi) f(\xi)\, d\xi.$$