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A note on the effectiveness of scheduled balanced ordering in a one-supplier two-retailer system with uniform end-customer demands

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ABSTRACT

For a decentralized supply chain with one supplier and two retailers that face uniformly distributed end-customer demands, a scheduled balanced ordering policy (SBOP) is one in which the two retailers take turns to order freely in one period of a two-period cycle, and receive a fixed shipment in the other period. We develop mathematical conditions, on the supplier and retailer cost parameters, that predict the effectiveness of the SBOP strategy in reducing total supply chain cost. We find that SBOP is often effective when the supplier has cost parameters larger than the retailers. We also show that SBOP can be effective even when there is no information sharing in the supply chain. Further, the effectiveness of SBOP is robust with respect to demand assumptions.

1. Introduction

Scheduled ordering policies were first studied by Lee et al. (1996) and Cachon (1999), albeit without the consideration of information sharing in the supply chain, and without mathematically examining the performance in terms of the total supply chain cost. A more rigorous analysis (along with a detailed discussion of their advantages) on the effectiveness of scheduled ordering policies can be found in Chen and Gavirneni (2010), where readers may also find other related literature. In this paper we have intentionally decided to keep the introduction short, and the reader can refer to Chen and Gavirneni (2010) paper for a detailed explanation on the practicality of the policies we analyze. The purpose of this short paper is to extend the work of Chen and Gavirneni (2010) to *include the scenario of no information sharing in the supply chain*, and to study the effectiveness of scheduled balanced ordering policy (SBOP) in reducing the total supply chain cost with *uniformly distributed end-customer demand*. The latter is to investigate whether the results in Chen and Gavirneni (2010) are robust with respect to demand assumptions.

We study a decentralized distribution supply chain with one supplier serving two retailers who face exogenous end-customer demands. The supplier and the two retailers are locally focused and try to minimize their own inventory-related costs. The end-customer demands the retailers face are *i.i.d.* over time, but can be correlated across the retailers. We refer to the demand the supplier faces as "retailer demand." Each retailer incurs a unit holding cost of h_r and a unit backorder cost of π_r. The supplier incurs a unit holding cost of h_s and a unit expediting (formally explained later) cost of π_s. There are no fixed ordering costs for the supplier and the two retailers. For such a setting, under the unrestricted *free ordering policy* (FOP), the retailers order freely in every period and they order up to the newsvendor solution. As an alternative, we propose a *scheduled balanced ordering policy* (SBOP), which operates with an ordering cycle of two periods. Under SBOP, the two retailers take turns to order freely in one period of a cycle, and receive a fixed shipment δ in the other period. In other words, in each period only one retailer can order freely and the other has to receive a predetermined quantity δ.

A detailed explanation as to why SBOP may be effective and why we study this particular policy can be found in Chen and Gavirneni (2010). Essentially, SBOP may achieve demand risk pooling for the supplier and enable her to achieve a cost reduction that is higher than the cost increase at the retailers, which leads to a reduction in total supply chain cost. This can be beneficial to everyone involved if the supplier is willing to share some of her savings with the retailers. Note that how the benefit is shared among the firms is beyond the scope of this research and we limit our attention to investigate under what circumstances SBOP can reduce the total supply chain cost so that all parties have incentives to implement the policy.

With total supply chain cost as the performance measure, we provide easy-to-evaluate conditions under which SBOP is effective in reducing the total supply chain cost. These conditions turn out

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to be similar to those in Chen and Gavirneni (2010), which proves the robustness of the results with respect to demand assumptions, and demonstrates that SBOP policies are more widely applicable. We also show that SBOP can be effective under certain scenarios even when there is no information sharing. This observation could be of significant managerial insight for firms that do not have incentive or technical capability to share information with their trading partners. Note that the conditions we provide are sufficient ones, SBOP thus can be more effective than the conditions specify. However, a simple numerical study shows that our conditions are quite robust and recommend the implementation of SBOP when it is most effective.

The rest of the paper is organized as follows. Section 2 introduces the model framework and provides the basic structural analysis. Section 3 compares the performance of FOP and SBOP under four scenarios and present sufficient conditions under which SBOP reduces the total supply chain cost. Section 4 presents a numerical study to illustrate the effectiveness of SBOP. We conclude the paper in Section 5.

2. Model setup and analysis

The sequence[1] of events in every period is as follows: (a) the supplier receives the units produced in the previous period; (b) the retailer who is free to order in that period places an order with the supplier; (c) if the supplier's inventory is not enough to satisfy all retailer demand, she obtains the product from an outside source immediately at a higher expediting cost, π_s; (d) the supplier ships the product to each retailer, either at the quantity the retailer ordered or δ; (e) the supplier decides the amount to be produced this period; (f) the end-customer demands occur and the retailers satisfy the demands as much as possible with their on-hand inventory. Unsatisfied demand at the retailers is backlogged; (g) inventory-related costs (holding costs for the supplier and the retailers, backorder cost for the retailers and expediting cost for the supplier) are tabulated.

In performing the analysis, we make the following assumptions[2]: (i) it takes the supplier one period to produce the product, which means one period of lead time; (ii) there's infinite capacity at the supplier; (iii) all retailer orders are fulfilled in every period. If the supplier does not have enough on-hand inventory, she will obtain the product from an outside source immediately (as in Gavirneni et al., 1999; Lee et al., 2000) at a higher expediting cost, and then ship it to the retailers right away. This implies a high service standard at the supplier side (As a real example, the Florida Dairy Marketing Cooperative (FDMC) that provides farm or unprocessed milk to fluid milk processors, buys unprocessed milk when it is unable to maintain optimal inventory levels from local member production Glenn et al., 2001.); (iv) the retailers receive their shipments immediately; (v) $\pi_r \geq h_r$ and $\pi_s \geq h_s$.

Let $D_{i,j}$ be the random end-customer demand that retailer i faces in period j of an ordering cycle, and $\mathcal{D}$ be a generic random variable referring to any $D_{i,j}$. $d_{i,j}$ is the realization of $D_{i,j}$. The supplier's demand (which comes from the retailers) is denoted by ξ. Also let $\Psi_Z(z)$ and $\psi_Z(z)$ be the c.d.f. and p.d.f. of a random variable Z. Define $\overline{\Psi}_Z(z)=1-\Psi_Z(z)$ and loss function $n_Z(R)=\int_R^\infty \overline{\Psi}_Z(z)\,dz$. The end-customer demands follow a uniform distribution. Without loss of generality, we assume the demand is defined on the range of [0,1] with expectation $\frac{1}{2}$ and c.d.f.

$$\Psi(z)=\begin{cases} z & \text{if } z\in[0,1], \\ 0 & \text{otherwise}. \end{cases}$$

Property 1 provides the c.d.f. of the sum of two *i.i.d.* uniformly distributed random variables.

Property 1. *Assume X_1 and X_2 are i.i.d. random variables with uniform distributions on the range of $[0,1]$ and $Z=X_1+X_2$. Then the c.d.f. of Z is*

$$\Psi_Z(z)=\begin{cases} \dfrac{z^2}{2} & \text{if } 0\leq z\leq 1, \\ -\dfrac{z^2}{2}+2z-1 & \text{if } 1\leq z\leq 2. \end{cases}$$

Proof. Follows from arguments based on convolution of two random variables. □

In the following Sections 2.1 and 2.2 we first introduce the models with information sharing. By information sharing we mean that the supplier knows all realized end-customer demands.

2.1. Free ordering policy

If a retailer orders in every period, it is optimal for him to use a stationary order-up-to policy. As a result, the end-customer demands will be transmitted to the supplier unaltered and it is optimal for the supplier to use a stationary order-up-to policy as well. It is well established that the optimal order-up-to level y_f^* for each retailer satisfies

$$y_f^*=\Psi_{\mathcal{D}}^{-1}\left(\frac{\pi_r}{h_r+\pi_r}\right)=\frac{\pi_r}{h_r+\pi_r}, \tag{1}$$

and the optimal order-up-to level Y_f^* for the supplier is

$$Y_f^*=\Psi_{\xi}^{-1}\left(\frac{\pi_s}{h_s+\pi_s}\right),$$

where ξ is the sum of two identical end-customer demands from the uniform distribution $\Psi_{\mathcal{D}}$. Since $\pi_s \geq h_s$, by Property 1 we can solve for Y_f^* as

$$Y_f^*=2-\sqrt{\frac{2h_s}{h_s+\pi_s}}. \tag{2}$$

2.2. Balanced ordering policy

Retailers' problem. Without loss of generality, we define the period that a retailer can order freely as the first period of a cycle for him. Assume that, at the beginning of any cycle, the retailer's inventory level after ordering is y_b. In the next period, he agrees to receive a fixed shipment δ. The one-cycle cost $l(y_b)$ for retailer i given an inventory level of y_b is

$$l(y_b)=h_r(y_b-D_{i,1})^+ +\pi_r(D_{i,1}-y_b)^+ +h_r(y_b+\delta-D_{i,1}-D_{i,2})^+ +\pi_r(D_{i,1}+D_{i,2}-\delta-y_b)^+.$$

The cost-to-go function for the remaining n cycles is

$$f_n(x)=\min_{y_b\geq x} v_n(y_b),$$
$$v_n(y_b)=E[l(y_b)+f_{n-1}(y_b+\delta-D_{i,1}-D_{i,2})].$$

Supplier's problem. At the beginning of every period (say t) when the supplier makes the production decision, she is anticipating a retailer demand at the beginning of next period $(t+1)$, which can be decomposed into two parts. The first part is the fixed shipment to the retailer that cannot order freely, δ. The second part is from the retailer that can order freely, which equals the

[1] Note that the sequence of events is the same as that in Chen and Gavirneni (2010). For ease of exposition, we reiterate here.
[2] The assumptions are also similar to those in Chen and Gavirneni (2010).