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Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment

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ABSTRACT

This study is motivated by the paper of Chang et al. [International Journal of Production Economics 123 (2010) 62–68]. We extend their inventory model from two aspects. (1) The demand rate as multivariate function of price and level of inventory (2) Delay in payment is permissible. Under these assumptions, we first formulated a mathematical model and then some useful theoretical results have been derived to characterize the optimal solutions for non-zero and zero ending inventory system. Numerical examples are presented to illustrate the theoretical results. The sensitivity analysis of a suitable example is performed and some managerial insights are proposed. Our analysis revels that it is more beneficial to keep higher inventory level, hoping to stimulate more demand, even if it results in non-zero ending inventory.

1. Introduction

The term "non-instantaneous deteriorating item" refers that an item maintains its quality or freshness for some extent of time and losses the usefulness from the original condition subsequently. In other words, for non-instantaneous deteriorating item deterioration does not occur prior to certain period of time. This characteristic can be usually observed in almost all food stuffs, fashionable items, electronics products etc. Wu et al. (2006) introduced the phenomenon "non-instantaneous deterioration" and established the optimal replenishment policy for non-instantaneous deteriorating item with stock dependent demand and partial backlogging. Subsequently, many researchers such as Ouyang et al. (2008), Wu et al. (2009), Maihami and Kamalabadi (2012a), Shah et al. (2013), Dye (2013) and references therein, have studied the inventory models for non-instantaneous deteriorating items under variety of conditions.

For an inventory system exhibiting a stock-dependent demand rate, the objective is to maximize the profit rather to minimize the costs as shown in Baker and Urban (1988). Moreover, as stated in Urban (1992), whenever the demand rate is influenced by displayed stock level; it is more profitable to keep higher inventory level, hoping to generate more demand, even if it turns up in non-zero ending inventory. Recently, some researchers such as Teng et al. (2011), Yang et al. (2011), Soni (2013) advocated these viewpoints and developed their model under different conditions.

Incorporating above points in model formulation for non-instantaneous deteriorating items, Chang et al. (2010), complemented the shortcomings of Wu et al. (2006) model to allow for (1) a profit-maximization, (2) a maximum inventory ceiling to reflect the facts that most retail outlets have limited shelf space or "too much piled up in everyone′s way leaves a negative impression on buyer and employee alike" as stated in Levin et al. (1972), (3) an ending-inventory to be nonzero when shortages are not desirable.

In aforesaid articles for non-instantaneous deteriorating items, it is assumed that a payment is made to the supplier immediately after receiving the items. However, in commercial practice, the supplier frequently offers certain credit period to the retailer to settle his/her debt without charging any interest. Offering such a credit period to the retailer, the supplier boosts the sales and reduces on-hand stock level. Simultaneously, without a primary payment the retailer can take the advantage of a credit period to accumulate the sales revenue and to earn interest on revenue. There are some interesting recent articles on trade credit such as Liao et al. (2012), Thangam (2012), Chern et al. (2013), Ouyang and Chang (2013), Zhong and Zhou (2013) and references therein. Due to significant practical relevance of trade credit policy, some researchers studied an inventory model for non-instantaneous deteriorating items under the condition of delay in payments. For instance, Ouyang et al. (2006) developed a model for non-instantaneous deteriorating items for finding the optimal ordering policy for a retailer facing constant demand and a supplier offering permissible delay in payments. Geetha and Uthayakumar (2010) further extended the work of Ouyang et al. (2006) by adding the shortage cost and the cost of lost sales. Soni and Patel (2012)

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presented a fuzzy framework to derive pricing and inventory policies with price dependent demand rate, wherein the authors extended the model of Geetha and Uthayakumar (2010) by characterizing holding cost rate, interest paid and interest earned rates as fuzzy variable. Musa and Sani (2012) considered different constant demand rates for before and after deterioration begins and determined ordering policies of delayed deteriorating items under permissible delay in payments. Maihami and Kamalabadi (2012b) discussed the pricing and lot-sizing problem for non-instantaneous deteriorating items, assuming time and price dependent demand rate, partial backlogging and lost sale case under permissible delay in payment.

The above-surveyed works for non-instantaneous deteriorating items under permissible delay in payment consider the demand function to be constant or price dependent or price and time dependent. Hence, we find that the inventory problem for non-instantaneous deteriorating items linked with trade credit has yet to be explored and understood especially when the demand rate be a multivariate function of price and level of inventory. In order to fill this gap, this paper tries to recast Chang et al. (2010) model to allow for delay in payments with price and stock-dependent demand rate. Under these considerations, we first formulate a mathematical model and then some useful theoretical results are derived to characterize the optimal solutions for non-zero and zero ending inventory system. Numerical examples are presented to illustrate the theoretical results. Sensitivity analysis and their observations carried out in the paper are attractive enough to receive attention on possible future characterization of the parameters.

The present paper is split into various sections such as introduction, assumptions and notations, model formulation, theoretical results, numerical examples, sensitivity analysis and conclusion including future enhancement of the model.

2. Assumptions and notations

The following notations and assumptions have been used in developing the mathematical model in this article.

2.1. Notations

A	The ordering cost per order.
M	Trade credit period.
c	The purchasing cost per unit.
p	The selling price per unit ($p > c$).
h	The inventory holding cost rate excluding interest charges rate.
Ip	The interest paid per dollar per unit time.
Ie	The interest earned per dollar per unit time.
t_d	The length of time in which the product has no deterioration.
T	Length of replenishment cycle (a decision variable).
q	The inventory level at time T (a decision variable).
θ	The deterioration rate of the on-hand inventory over $[t_d, T]$.
Q	The inventory level at time 0.
U	The maximum inventory level.
$I_1(t)$	The inventory level at time t ($0 \le t \le t_d$) in which the product has no deterioration.
$I_2(t)$	The inventory level at time t ($t_d \le t \le T$) in which the product has deterioration.

2.2. Assumptions

(1) The inventory system involves single non-instantaneous deteriorating item.
(2) Replenishment rate is infinite and lead time is zero.
(3) Shortages are not allowed to avoid lost sales.
(4) The maximum inventory level is U to reflect the facts that most retailer outlets have limited shelf space, and "too much piled up in everyone's way leaves negative impression on buyer and employee alike" (cf. Chang et al.; 2010, Teng et al.; 2011)
(5) The initial and ending inventory level are not restricted to be zero. To make the replenishment cycle repeatable into the future, we assume that the initial and ending inventory levels are the same. (cf. Teng et al.; 2011)
(6) Demand rate $D(p,t)$ is a function of the selling price and current inventory level. We assumed that $D(p,t) = \alpha(p) + \beta I(t), 0 \le t \le T$, where $\beta > 0$ is stock dependent parameter and $\alpha(p)$ is positive decreasing function of p with $a'(p) = \mathrm{d}\alpha(p)/\mathrm{d}p < 0$.
(7) During the fixed period, t_d, the product has no deterioration. After that the on-hand inventory deteriorate with constant rate θ, where $0 < \theta < 1$. For simplicity, we assume that t_d is given constant and $t_d \le T$.
(8) There is no replacement or repair of deteriorated units during the period under consideration.
(9) During the trade credit period, M, the account is not settled; the revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off the item ordered, and starts to pay the interest charged on the item in stock.

3. Model formulation

An order $Q-q$ units arrives at time $t=0$. Since the ending inventory is q units, the inventory level at time $t=0$ is Q units. The inventory level is declining only due to demand rate over time interval $[0, t_d]$. After that the inventory gradually depletes to q (≥ 0) owing to demand and deterioration during the time interval $[t_d, T]$. The process is repeated as mentioned above (see Fig. 1).

From the above assumptions and notation, the status of inventory at any instant of time $t \in [0, T]$ is governed by differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \begin{cases} -[\alpha(p) + \beta I_1(t)] & 0 \le t \le t_d \\ -\theta I_2(t) - [\alpha(p) + \beta I_2(t)] & t_d \le t \le T \end{cases} \quad (1)$$

with boundary condition $I(0) = Q$ and $I(T) = q \ge 0$. The solution of Eq. (1) is

$$I(t) = \begin{cases} I_1(t) \text{for} 0 \le t \le t_d \\ I_2(t) \text{for} t_d \le t \le T \end{cases} \quad (2)$$

where

$$I_1(t) = \left(Q + \frac{\alpha(p)}{\beta}\right) e^{-\beta t} - \frac{\alpha(p)}{\beta} \quad (3)$$

$$I_2(t) = \left(q + \frac{\alpha(p)}{\eta}\right) e^{\eta(T-t)} - \frac{\alpha(p)}{\eta}, \text{where } \eta = \beta + \theta \quad (4)$$

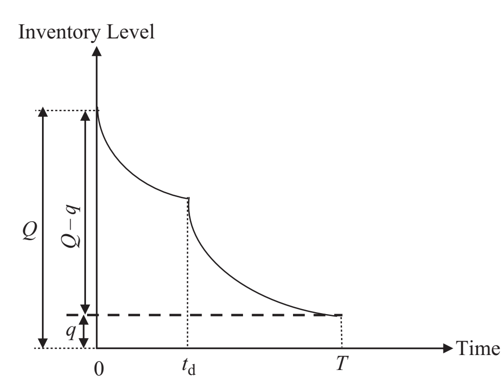

Fig. 1. Inventory profile when q is positive.