Int. J. Production Economics 146 (2013) 313–324

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal algorithm for the demand routing problem in multicommodity flow distribution networks with diversification constraints and concave costs

Pablo Cortés*, Jesús Muñuzuri, José Guadix, Luis Onieva

Ingeniería de Organización. Escuela Técnica Superior de Ingeniería. University of Seville c/Camino de los Descubrimientos, s/n E-41092 Sevilla, Spain

ARTICLE INFO

Article history:
Received 6 April 2011
Accepted 18 July 2013
Available online 2 August 2013

Keywords:
Demand routing
Concave cost
Distribution network
Kuhn Tucker conditions

ABSTRACT

Distribution problems are of high relevance within the supply chain system. In real life situations various different commodities may flow in the distribution process. Furthermore, the connection between production and demand centres makes use of complex mesh networks that can include diversification constraints to avoid overcharged paths. In addition, the consideration in certain situations of economies of scale gives rise to non-linear cost functions that make it difficult to deal with an optimal routing scheme. This problem is well represented by the multicommodity flow distribution networks with diversification constraints and concave costs (MFDCC) problem. Here we present an optimal algorithm based on the Kuhn–Tucker optimality conditions of the problem and capable of supplying optimal distribution routes in such complex networks. The algorithm follows an iterative procedure. Each iteration constructive solutions are checked with respect to the Kuhn–Tucker optimality conditions. Solutions consider a set of paths transporting all the demand allowed by its diversification constraint (saturated paths), a set of empty paths, and an indicator path transporting the remaining demand to satisfy the demand equation. The algorithm reduces the total cost in the network in a monotonic sequence to the optimum. The algorithm was tested in a trial library and the optimum was reached for all the instances. The algorithm showed a major dependency with respect to the number of nodes and arcs of the graph, as well as the density of arcs in the graph.

1. Introduction

Distribution problems are of high relevance within the supply chain system. One of the principal root problems in the family is the transportation problem that was introduced in 1941 by Hitchcock (1941). This has been researched extensively due to its importance and the wide range of applications it has. The transportation problem involves a network with a group of production centres willing to send their commodity to a group of demand centres through a set of arcs with linear costs.

However, in real life situations various different commodities may flow instead of just one commodity (e.g. Kengpol et al., 2012). Furthermore, each production centre is not connected with each demand centre using a single arc but rather with a mesh network of different arcs (Ubeda et al., 2011). Certain cases cannot be correctly modelled using a linear cost function and require a more complex way of modelling (Tsao and Sheen, 2012). Sometimes more accurate results can be achieved with concave cost functions. This situation arises when dealing with economies of scale which are very common in many fields such as freight distribution and demand routing systems, and even other technological fields such as telecommunication networks.

In this paper, we consider a mesh network with a set of nodes as the origin of demand and a set of nodes as the demand destination. Each pair of nodes constitutes an origin-destination pair which is known as a specific commodity, converting the problem into a multicommodity flow problem. Additionally, the costs in arcs are modelled by concave costs. Lastly, we propose the diversification constraints needed to prevent the arcs from becoming extremely loaded which can produce malfunction effects on the network and may arise when concave costs are involved as a result of the benefits offered by economies of scale. This type of constraint increases the survivability and reliability of the networks.

Distribution problems with concave costs are NP-Hard and their optimal solution cannot be found in polynomial time. Several authors have proposed approximate methods to tackle this problem, such as the linear approximation by Thach (1992), the lagrangian relaxation by Larsson et al. (1994), or the dynamic programming approach by Zangwill (1968), Florian and Klein

* Corresponding autor. Tel.: +34 954486153.
E-mail address: pca@us.es (P. Cortés).

0925-5273/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2013.07.016

(1971) and Burkard et al. (2001). Metaheuristic approaches have also been tested although mainly in telecommunication network problems (Kapsalis et al, 1993; Dengiz et al, 1997; Altiparmak et al, 2003; Zhou and Gen, 2003). In a more general case, Yan and Luo (1999) developed a heuristic based on simulated annealing and threshold acceptance, and Altiparmak and Karaoglan, (2008) proposed a tabu approach to solve the transportation problem with concave costs. Here, we have developed an optimal approach based on the optimality Kuhn–Tucker conditions of the mathematical formulation of the problem.

The paper then defines the problem and its mathematical formulation in Section 2. Section 3 presents the Kuhn–Tucker optimality conditions for the demand routing problem in multicommodity flow networks with diversification constraints and concave costs that has been described in the previous section. Next, the solution based on the optimality constraints demand routing algorithm is proposed is Section 4. The results for a trial library with 27 generated instances are presented in Section 5 and the detailed procedure for an example network is developed. Finally, the main conclusions are presented in Section 6.

2. Problem definition and formulation

Given a graph $G=(N,E)$ where N is the set of nodes and E the set of arcs, a set of commodities K is considered and represents every origin-destination pair of demand. We define each arc of the network as $e \in E$ and the set of paths connecting every origin-destination pair is given by $P(k)$. A subset of this set is given by the arc-disjoint paths, $P^d(k)$.

Let γ_k be the demand volume for each origin-destination pair, k, and δ_k the diversification parameter for each pair k, so that the demand of pair k is routed along $\lceil 1/\delta_k \rceil$ different paths. As paths and arcs are pre-computed, φ_{eh} is a binary parameter that determines if arc e is on path h.

The variables of the problem are the demand fraction related to the origin-destination pair k that is routed on path h, which is a continuous variable given by p_{kh}, and the total flow transported on arc e, including the total amount of flow from all the commodities on the arc, which is a continuous variable given by x_e.

Owing to the nature of the problem described, numerous concave cost functions can be considered. We used the objective function given by $c_e(x_e)=d_e\sqrt{x_e}$ for arc e, where x_e has been previously explained and d_e means the per unit variable cost corresponding to the arc. We selected this function because it provides a widely acknowledged basis of analysis of scale economies' effect in transportation problems, and because it is taken as a representative cost function very similar to many transportation cost functions in practice (see detailed arguments in LeBlanc, 1976; Larsson et al., 1994; Yan and Luo, 1999; and Altiparmak and Karaoglan, 2008). The cost concave function is, therefore, a differentiable function. Note that the objective function is used to provide comparable numerical examples in the subsequent results section. The selection of this function does not condition the methodology and developed algorithm, because the objective function only affects the procedure for the calculation of the derivative providing numerical data.

As a result, the demand routing problem in multicommodity flow networks with diversification constraints and concave costs (MFDCC problem) can be formulated as:

$$\text{Minimize} \quad \sum_{e \in E} c_e(x_e)$$

s.t.:

$$x_e = \sum_{k \in K} \sum_{h \in P(k)} \gamma_k \phi_{eh} p_{kh}, \quad \forall e \in E \tag{1}$$

$$\sum_{h \in P(k)} p_{kh} = 1, \quad \forall k \in K \tag{2}$$

$$p_{kh} \le \delta_k \,, \quad \forall h \in P(k), \quad k \in K$$
$$x_e \ge 0, p_{kh} \ge 0 \tag{3}$$

Constraint (1) calculates the total amount of flow on the arc e; constraint (2) ensures that the demand for every origin-destination pair is met; and constraint (3) ensures that the demand is routed along $\lceil 1/\delta_k \rceil$ alternative paths. Set $P^d(k)$ represents all the feasible disjoint paths for each origin-destination pair. The "disjoint" concept is important and can be weakly imposed on the arcs or strongly imposed on the nodes. Here, we have chosen the arc diversification concept because in most cases it is normally only necessary to take this weak constraint into consideration. Using an example from telecommunications (Cortés et al., 2001), disjoint arcs can be used to model reality, since the optical cross-connect (OXC) in an all-optical WDM mesh network is seldom broke. The OXC is a device which switches an optical signal from an incoming fibre to an outgoing fibre on the same wavelength. So, constraint (3) is equivalent to

- $p_{kh} \le \delta_k, \ \forall (i,j) \in h \,, \ h \in P^d(k), \ k \in K$, constraint for arc-disjoint paths
- $p_{kh} \le \delta_k, \forall j \in h, \ h \in P^d(k), \ k \in K$, constraint for node-disjoint paths

Depending on what is being considered and as explained above, we are looking at constraints for arc-disjoint paths.

3. Kuhn Tucker conditions for the demand routing problem in multicommodity flow networks with diversification constraints and concave costs

Every solution verifying the necessary and sufficient Kuhn–Tucker conditions of any problem will be the optimal solution of problem. This fact is proven for linear or non-linear problems since they are global optimality conditions. Additionally, duality theory can be regarded as a particular case of the Kuhn–Tucker conditions for linear problems. See classic theory such as Hillier and Lieberman (1974), Nocedal and Wright (2006), Avriel (2003) for detailed explanations on Kuhn–Tucker conditions. Appendix A depicts a general overview on the Kuhn–Tucker necessary optimality conditions and their detailed corresponding application to MFDCC problem, and Appendix B depicts the equivalent general overview and detailed application of the Kuhn–Tucker sufficient optimality conditions to the MFDCC problem.

Next, we re-formulate MFDCC problem to specify the objective function in terms of p_{kh} variables. This can be done because p_{kh} and x_e are inter-related by constraint (2). Therefore, constraint (2) is included in the objective function to re-formulate the problem and subsequently formulate the Kuhn–Tucker necessary optimality conditions

$$\text{Minimize} \sum_{e \in E} c_e(x_e) \equiv \text{Minimize } C(x) \equiv \text{Minimize } C(p)$$

s.t. :

$$\gamma_k - \sum_{h \in P(k)} p_{kh} = 0, \quad \forall k \in K \leftarrow \mu_k \text{ (free multiplier)}$$

$$p_{kh} - \delta_k \cdot \gamma_k \le 0 \,, \quad \forall h \in P^d(k), \quad \forall k \in K \leftarrow \nu_{kh} \text{ (non} - \text{negative multiplier)}$$

$$-p_{kh} \le 0, \quad \forall h \in P^d(k), \quad \forall k \in K \leftarrow u_{kh} \text{ (non} - \text{negative multiplier)}$$

Kuhn–Tucker conditions state that all $h \in P^d(k)$ and $k \in K$ can be written as:

$$\frac{\partial C(p)}{\partial p_{kh}} - \mu_k = u_{kh} - \nu_{kh}$$