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A multi-round partial beam search approach for the single container loading problem with shipment priority

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ABSTRACT

We consider shipping priority in the single container loading problem, where all boxes with high priority must be loaded into the container before those with low priority. Shipping priority is a very common consideration in real applications, but it has received very little attention from the research community. We propose a multi-round partial beam search method that explicitly considers shipping priority when evaluating the potential of partial solutions to solve this problem. Experiments on existing benchmarks suggest that our approach is more effective than current methods. The average utilization is improved by almost 1% while the running time is shorter than the state of the art method. Since existing benchmark data covers only weakly heterogeneous instances, we extend the benchmark data to strongly heterogeneous instances and also generate instances with various proportions of high priority boxes to cover a wider spectrum of applications.

1. Introduction

The use of standard containers to ship products has grown tremendously in recent years. According to the 2011 annual review by the liner shipping information platform *Alphaliner*, the cellular container ship capacity will grow by an average annual rate of 8.7% between 2011 and 2012, corresponding to an increase of 1.25 million 20-ft equivalent units (TEU) in 2011 and 1.33 million TEU in 2012. Hence, even a single percent increase in volume utilization will translate to large savings.

The Single Container Loading Problem (SCLP) has received considerable attention in recent years. It is commonly modeled as orthogonal packing of rectangular objects. Given a set of 3D rectangular boxes, the task is to orthogonally load the boxes into a 3D rectangular container without overlap so that the total utilization of the container is maximized. According to the typology for cutting and packing problems proposed by Wäscher et al. (2007), the SCLP is classified as a three-dimensional Single Large Object Placement Problem (3D-SLOPP) when the boxes are weakly heterogeneous (i.e., there are only a few box types) or as a three-dimensional Single Knapsack Problem (3D-SKP) when the boxes are strongly heterogeneous (i.e., there are many box types).

The Single Container Loading Problem with Shipment Priority (SCLP-SP) is a useful variant of the SCLP. Each box to be loaded is designated as either high priority or low priority, and all high priority boxes must be loaded in the final solution. In the real world, this type of shipment priority exists in many circumstances when the available container space is limited. For example, perishable goods such as fresh fruit should be delivered before non-perishable goods like canned products. Also, different customer orders have different deadlines, so companies will deliver the earlier orders first to avoid incurring penalties. A carrier might also prioritize the goods of long-term customers ahead of those of new customers in order to reward customer loyalty. Despite its applicability to a variety of practical scenarios, the SCLP-SP has not yet received significant attention from the research community; to the best of our knowledge, the article by Ren et al. (2011) is the only work examining this specific problem in the literature.

There are three main contributions of this work. Firstly, we propose a multi-round partial beam search algorithm for the SCLP-SP. It can be classified as a block building approach, which has been proven effective for SCLP. The main advantage of using beam search is that a set of promising states is kept at each level instead of one state. It is well known that strongly heterogeneous SCLP instances are challenging; with shipping priority, such instances become even more challenging, and in some cases even finding a feasible solution is hard. In such a situation, keeping a variety of solutions becomes a useful tool to increase the odds of finding feasible solutions. Secondly, we explicitly consider shipping priority when evaluating the potential of partial solutions. It is well known that the success of a meta-heuristic greatly depends on the accuracy of the fitness measure. Experiments confirm that our new measure is more effective than existing measures. Thirdly, we generate new test data. The benchmark data proposed by Ren et al. (2011) covers only

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weakly heterogeneous instances and the proportion of high priority boxes is exactly 50%. We extend the benchmark data to strongly heterogeneous instances and generate instances with various proportions of high priority boxes. The new test set covers more application scenarios, and also includes "more difficult" instances.

The rest of the paper is organized as follows. Section 2 presents an overview of existing work that is related to the SCLP-SP, and we formally define the problem in Section 3. We then describe our proposed multi-round partial beam search (MultiRoundPartialBeamSearch) approach in the next four sections, where we explain how we generate blocks of boxes that are used as primitives in our search process (Section 4); the concepts of state and state transition (Section 5); the partial beam search procedure (Section 6); and our overall multi-round partial beam search framework (Section 7). In Section 8, we discuss practical considerations related to the deployment of our approach. The experiments that we have conducted to evaluate the effectiveness of MultiRoundPartialBeamSearch are described in Section 9. We conclude our paper in Section 10 with some closing remarks.

2. Literature review

The single container loading problem is one of the classic problems in the field of cutting and packing. Since the decision version of the SCLP is NP-complete (Pisinger, 2002), existing research has focused on heuristic methods in order to solve instances of practical size. These heuristic methods can be roughly divided into three groups: simple heuristics, meta-heuristics and tree search methods.

The earliest proposed heuristic techniques are *simple heuristics*. Bischoff and Marriott (1990) performed a comparative analysis of fourteen heuristic rules. Bischoff and Ratcliff (1995) proposed selecting boxes according to high utilization layers when stability was considered, and using a four-tier ranking scheme for a multi-drop situation. Although simple heuristics are time-efficient, they generally produce poorer solutions than meta-heuristics and tree search approaches when sufficient computation time is available.

Many *meta-heuristics* have been used to solve the SCLP, of which genetic algorithms are the most common (Bortfeldt and Gehring, 2001; Techanitisawad and Tangwiwatwong, 2004; Yeung and Tang, 2005; Gehring and Bortfeldt, 1997; Gonçalves and Resende, 2012). Other meta-heuristic approaches for the SCLP include the sub-volume based simulated annealing method proposed by Jin et al. (2004), the GRASP methods devised by Moura and Oliveira (2005) and Parreño et al. (2008), as well as the variable neighborhood search by Parreño et al. (2010). *Tree search methods* have also been applied to the SCLP (Pisinger, 2002; Eley, 2002; Wang et al., 2008; Christensen and Rousøe, 2009; Fanslau and Bortfeldt, 2010). In these work, simple heuristics are usually used when evaluating the branches of the search tree. In addition, heuristics can also be used as the main framework while tree search act as an auxiliary role. For example, Zhu and Lim (2012) devised a greedy heuristic in which the evaluation function is performed by a two-depth tree search.

Block-based method is the most popular method when dealing with the container loading problem. Blocks are a set of several boxes. Many previous approaches utilize blocks that are composed of only a single type of box in the same orientation (Eley, 2002; Mack et al., 2004; Parreño et al., 2008), while Fanslau and Bortfeldt (2010) and Zhu and Lim (2012) employed blocks that can contain different boxes in different orientations. Other types of blocks have also been proposed. George and Robinson (1980) introduced a wall-building approach that filled the container with vertical layers called "walls"; this approach was later adopted by Bortfeldt and Gehring (2001) and Moura and Oliveira (2005). George (1992) also performed packing using walls in a local search approach that rebuilt and rearranged layers. Pisinger (2002) decomposed the SCLP into a number of walls, and then further divided the walls into strips before solving the packing problem for each strip optimally by treating them as knapsack problems. When the walls are horizontal, the corresponding approach is usually called layer-building; George and Robinson (1980) proposed a layer-building approach in which the container was filled by building layers across its width and then combining the space left over. Bischoff and Ratcliff (1995) applied this method when generating stable packing patterns. In addition, Gehring and Bortfeldt (1997) and Huang and He (2009) devised stack-building approaches that built towers of boxes, and Lim et al. (2003) proposed a multi-faced approach that built packing patterns from every wall of the container without requiring flat layers.

Practical constraints are considered in the study of the SCLP. Bischoff and Ratcliff (1995) summarized a spectrum of situations encountered in practice and divided them into twelve different constraints, some of which have been more heavily studied than others. For example, *orientation* constraints determine the permitted orientations of the boxes when they are loaded. Other than the extreme cases where all of the six possible orientations are allowed or no rotation is allowed, some applications allow only some of the possible orientations (e.g., the loading of major appliances that should only be stored upright). Mack et al. (2004) and Moura and Oliveira (2005) handled such orientation constraints in their work. The *shipment priority* constraint, where the boxes to be loaded are either high or low priority, and all high priority boxes must be loaded, was also mentioned by Bischoff and Ratcliff (1995). However, it has been largely ignored in the existing literature. To the best of our knowledge, only Ren et al. (2011) have studied this constraint.

There are several proposed *load stability* constraints in existing research that handle the possibility of shifting or toppling boxes during shipping that may cause damage to the goods. For certain applications, there is a requirement that all loaded boxes must be entirely supported from below, either by the floor of the container or by the upper surfaces of other boxes. This is known as the full-support variant (Bischoff and Ratcliff, 1995), and SCLP versions without this constraint are called non-support variants. The approach by Fanslau and Bortfeldt (2010) is applicable to both full-support and non-support situations. Mack et al. (2004) discussed another variant that considered only partial support such that the total supporting area of each box from below must be above a certain threshold, or the center of gravity of the box must be supported from below. Another kind of stability caters for lateral support, such that the four side faces of a stowed box must be adjacent to either the container or other boxes. This arrangement can effectively avoid shifting during transportation (Eley, 2002).

To our knowledge, Zhu and Lim (2012) are currently the best algorithm for the container loading problem without priority consideration. In our algorithm, we have adopted several elements from Zhu and Lim (2012): the block generation method; the free spaces representation; free spaces ranking criteria; and the placement of blocks. The difference between these two algorithms is twofold: search framework and the evaluation of the fitness. In their work, they proposed a greedy approach to build only one solution; the method proposed in this paper is a partial beam search method, which maintains the diversity of the paths explored. In Zhu and Lim (2012), the block selection and solution evaluation emphasize the utilization of the container; while in our algorithm, we consider more about high priority boxes when selecting blocks and evaluating solutions. Actually, their work is not applicable to problems with the priority constraint.

3. Problem description

The Single Container Loading Problem with Shipping Priority (SCLP-SP) is formally defined as follows. We are given a