Int. J. Production Economics 145 (2013) 724–732

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

The equilibrium quantity and production strategy in a fuzzy random decision environment: Game approach and case study in glass substrates industries

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ARTICLE INFO

Article history:
Received 13 June 2012
Accepted 16 May 2013
Available online 11 June 2013

Keywords:
Fuzzy random variable
Game theory
Equilibrium quantity
Entropy
Case study

ABSTRACT

This paper develops a two-stage Cournot production game that integrates strategic and operational planning under the fuzzy random environment, which to our best knowledge has not appeared in the literature. At the strategic level, two competing decision-makers determine the upper bound of a production quantity under a high-production strategy and the lower bound of the production quantity under a low-production strategy. Then at the operational level, the two competitors determine the range-type production quantity that is assumed to be a triangular fuzzy number represented by the apex and the entropies rather than a crisp value. The apex of a fuzzy equilibrium quantity can be obtained by the conventional Cournot game as the membership value is equal to one. A fuzzy random decision can be represented by entropies derived from the fuzzy random profit function of each firm in a specific production strategy. A case study of two leading firms in the glass substrates industry demonstrates the applicability of the proposed model. The finding that both firms would tend to adopt the common strategy coincides with observed real-world behavior. We conclude that our proposed method can provide decision-makers with a simple mathematical foundation for determining production quantity under a production strategy in a fuzzy random environment.

1. Introduction

Decision-making in complicated and competitive environments can be a difficult task because of uncertainty in the form of *ambiguity* or *randomness*. After the notion of fuzzy sets theory was introduced by Zadeh (1965) to manage ambiguity, the theory underwent extensive development and today is routinely applied to solve a variety of real-world problems, (Kunsch and Fortemps, 2002; Tanaka, 1987; Wong and Lai, 2011). Problems of randomness can be properly modeled by probability theory; applications to real problems appear in (Pastor et al., 1999; Valadares Tavares et al., 1998; Zhang et al., 2004). However, decision-makers often work in a hybrid (uncertain) environment where ambiguity and randomness exist simultaneously. Given such environments, a fuzzy random variable as introduced by Kwakernaak (1978) is a useful tool for solving these two aspects of uncertainty. Other studies (Colubi et al., 2001; Krätschmer, 2001) also extend several theories (Wang et al., 2007; Wang et al., 2008) to environments with these two aspects of uncertainty.

The typical Cournot game (see Cournot, 1838) models a duopoly in which two competing firms choose their production quantity. In the equilibrium quantity, no firm can be better off by a unilateral change in its solution. The exact values of parameters are required information when the Cournot game is applied to decision-making models, but exact values are often unobtainable in a business environment. Yao and Wu (1999) probably initiated a non-cooperative game involving fuzzy data by applying the ranking method to defuzzify the fuzzy demand and fuzzy supply functions into crisp values such that both consumer surplus and producer surplus can be calculated in a conventional manner. Their method of transforming fuzzy numbers to crisp values is also utilized to construct the monopoly model in Chang and Yao (2000). Liang et al. (2008) propose a duopoly model with fuzzy costs to obtain the optimal quantity of each firm. Recently Dang and Hong (2010) propose a fuzzy Cournot game with rigorous definitions ensuring a positive equilibrium quantity and with a flexible controlling mechanism that adjusts the parameters of associated objective functions. As mentioned, the resulting crisp values derived by previous studies are counter-intuitive outcomes of the problem in the fuzzy sense. A crisp decision is too precise

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http://dx.doi.org/10.1016/j.ijpe.2013.06.002

to be believed in a business environment with a fuzzy sense. To bridge this gap, we solve for a range-type solution rather than a crisp-value one. In addition, decision-makers may prefer a decision with more information in a range they themselves can adjust. In this case study, we empirically demonstrate our proposed approach to investigate the behaviors of the equilibrium quantity and production strategy in the glass substrates industry, which is highly volatile and ambiguous in the market demand and production costs.

In reality, the parameters of the Cournot game may behave with the characteristic of randomness in nature. In a business environment, decision-makers may predict demand behavior in a market, i.e. a functional form describes the relationship between price and quantity. The parameters of the functional form of market demand typically can be estimated by the technique of econometrics. Regression is one of the most popular statistical approaches employed and leads to game-theoretical models with random parameters. Hosomatsu (1969) indicates that the Cournot solution to an oligopolistic market is based upon the implicit assumptions given the estimated market demand function. Sato and Nagatani (1967) propose a model to relax the Cournot assumption and substitute firms' subjective evaluation of a market with a more general form with randomness.

Unpredictable events may drive the price fluctuation over a short period. It is appropriate to apply the fuzzy regression method (Chen and Dang, 2008; González-Rodríguez et al., 2009) to manage the data with ambiguity sense. The resulting estimators derived by econometrics involve in ambiguity and randomness, which occur in many fields (Guo and Lu, 2009; Xu and Zhao, 2010). In fact, Guo and Lu (2009) state that the coexistence of ambiguity and randomness becomes an intrinsic characteristic in the real world. Thus, there is a strong motivation to develop the Cournot production game with parameters of ambiguity and randomness described by fuzzy random parameters. To our best knowledge, none of or very little research involves in such a Cournot production game.

The Cournot production game proposed in this paper incorporates a production strategy which refers to the pattern of production quantities chosen by a decision-maker. Facing uncertainty with ambiguity and randomness, the choice of production strategy has important consequences for the selection, deployment and management of production resources. In general, there are two major stages of decision sequence: strategic and operational levels (see Ballou (1992)). Many studies focus only on the strategic level or operational level and ignore the importance of the interaction between them. In this paper, we propose a model considering both strategic and operational levels. Our model is similar to other two-stage games (see (Bae et al., 2011; Dhaene and Bouckaert, 2010)) where a player's decision in the first stage affects the action taken in the second stage. We consider two specific types of production strategies – high and low – by which the decision-maker's profit function depends on the highest production quantity or the lowest production quantity, respectively. In the long term, a high- or low-production strategy may be employed because the decision-maker desires to gain market share or to enhance the quality of products (Stout, 1969; Walters, 1991). Furthermore, Yang and Wee (2010) indicate that the production strategy is needed to respond to the market demand because of the rapid technology change (see (Droge et al., 2012; Kenne et al., 2012; Xu et al., 2012) for other models adopting strategy perspectives). The aim of this paper is to solve for the fuzzy equilibrium quantity of each decision-maker and to provide an appropriate production strategy under the fuzzy random business environment.

The remainder of this paper is organized as follows. Section 2 presents the preliminary knowledge of the fuzzy sets theory, entropy, and fuzzy random variable. Section 3 addresses the Cournot game in the fuzzy random environment and solves for the fuzzy equilibrium quantity of each firm. Section 4 investigates the insights of the proposed method including the extension and discussion. Section 5 illustrates the applicability of the proposed model in the real-world situation. Section 6 discusses our conclusions and gives suggestions for future research.

2. Definitions and concepts

This section introduces the fuzzy sets theory, entropy and expected operator which are integral to this paper.

2.1. Fuzzy sets theory

The fuzzy sets theory initiated by Zadeh (1965) attempts to analyze and solve problems with a source of ambiguity called fuzziness. In the following, we introduce the definitions and notations of triangular fuzzy numbers, α-level cut and the extension principle.

2.1.1. Triangular fuzzy number

For practical purposes, one of the most commonly used fuzzy numbers is the triangular type because it is easy to handle arithmetically and has an intuitive interpretation (Dağdeviren and Yüksel, 2008; Şen et al., 2010). Giannoccaro et al. (2003) and Petrovic et al. (1999) show that triangular fuzzy numbers are the most suitable for modeling market demand in the fuzzy sense (see (Ayağ and Özdemir, 2012; Vijay et al., 2005) for other applications of triangular fuzzy numbers). The membership function $\mu_{\tilde{A}}(x)$ of a triangular fuzzy number $\tilde{A}$ can be defined by

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-m_A+l_A}{l_A}, & m_A-l_A\le x\le m_A\\ \frac{m_A+r_A-x}{r_A}, & m_A\le x\le m_A+r_A\\ 0, & \text{otherwise,}\end{cases} \tag{1}$$

where $\tilde{A}$ is represented as a triplet (m_A-l_A, m_A, m_A+r_A) and m_A, l_A and r_A are the apex, left and right spreads of the fuzzy number $\tilde{A}$, respectively. Furthermore, a triangular fuzzy number $\tilde{A}$ can be shown in Fig. 1.

2.1.2. α-Level cut

One of the most important concepts of fuzzy sets is the α−level cut given by

$$\tilde{B}_\alpha := \{x\in\mathbb{R}|\tilde{B}(x)\ge\alpha\}$$

where $\alpha\in[0,1]$, which means for a fuzzy number $\tilde{B}$, those elements whose membership values are greater than or equal to α.

2.1.3. Extension principle

Let "$\odot$" be any binary operation $\oplus$ and $\otimes$ between two fuzzy numbers $\tilde{A}$ and $\tilde{B}$. Based on the extension principle, the membership function of $\tilde{A}\odot\tilde{B}$ is defined by

$$\mu_{\tilde{A}\odot\tilde{B}}(z)=\sup_{x\circ y}\min\{\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(y)\}$$

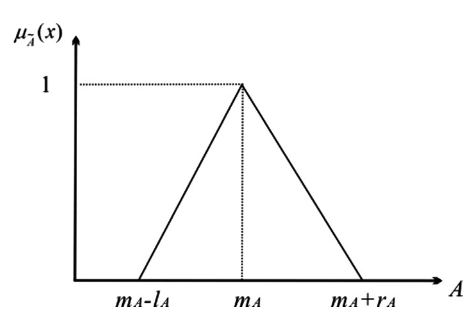

Fig. 1. A triangular fuzzy number $\tilde{A}$.