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Optimal inventory policy for a perishable item with demand function sensitive to price and time

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ABSTRACT

We formulate a model for determining the optimal pricing, order quantity and replenishment period for perishable items with price-dependent and time-dependent demand. The items have a fixed shelf-life, and the demand rate decreases linearly in the selling price and polynomially over the time after replenishment, until it vanishes either at the reservation price or at expiration time. We prove that the three-variable profit maximization problem can be reduced into a single-variable problem, in which the variable is the duration of the replenishment period. We show that the profit function is strictly pseudo-concave and provide means of obtaining the optimal policy. Three numerical examples are presented to demonstrate the model accompanied by a sensitivity analysis.

1. Introduction

The primarily negative relationship between the price of a product and the quantity demanded is well established (see Kocabiyikoglu and Popescu (2011) for examples of some common price-dependent demand functions). Yet, in addition to price, there are several other marketing-related factors that may affect the demand for a product at a given time (hereafter referred to as the "demand rate"). Bose et al. (1995) developed a deterministic model for an item whose demand follows a linear trend over time. Wu et al. (2006), Chang et al. (2010) and Devangan et al. (2013), following Baker and Urban (1988), considered also the influence of the quantity on-hand on retail product demand (an effect associated with the impact of shelf-space allocation). Avinadav and Arponen (2009) suggested a demand rate that is positively affected by the remaining shelf-life of the product, since higher time-to-expiry indicates better freshness and higher quality.

In this work we focus on finding optimal pricing and inventory policies while considering the effect of two marketing motivators: price and remaining shelf-life duration. While the effect of price on demand has been thoroughly analyzed in the literature (see, for example, Petruzzi and Dada (1999) and several more examples in the next section), there is a paucity of research on the effect of the remaining shelf-life duration despite its importance in modeling the demand rate of perishable items (including most grocery items, such as dairy products, bread, beverages and many other industrial food products, as well as ammunition, batteries and printer ink). In general, the age of inventory is likely to negatively affect the demand for perishable goods. Sarker et al. (1997) claim that this effect occurs because consumers tend to feel less confident purchasing perishable items whose expiration dates are approaching. Some items, such as range ammunition, suffer a negligible decrease in demand up to their expiration date, since they are used immediately after being purchased. On the other hand, the demand for self-defense ammunition is sensitive to the inventory's remaining shelf-life, since it is (hopefully) less likely to be used before its expiration.

Our approach seeks to contribute toward the efficient management of perishable inventory and the prevention of waste. Waste is a key source of food loss in supply chains and in inventory systems with perishable items, and it is influenced by managerial factors as well as by technological factors, including production facilities, preservative materials, packaging, storage, and transportation conditions, all of which are vital for maintaining a product's shelf-life. Prevention of food waste is of particular interest in the discussion of perishable inventory management, as perishable food products are a main source of profit for grocery retailers.

According to Donselaar and Broekmeulen (2012), the United States Department of Agriculture (USDA) estimates that average annual food losses in supermarkets in 2005 and 2006 were 11.4% for fresh fruit, 9.7% for fresh vegetables and 4.5% for fresh meat, poultry and seafood. Global trends indicate a future further increase of waste. A possible explanation of this trend is the increasing number of supermarkets (Parfitt et al., 2010), as well as the growth of household incomes, which increases the demand for perishable products (e.g., Parfitt et al. (2010) indicate that the

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demand for meat in China is increasing rapidly). Retailers incur heavy costs for wasted goods, as in addition to lost sales, they must bear the disposal costs of expired items.

This paper proposes a model that can be used as a tool in the management of perishable inventory in general and food products in particular, enabling retailers to compare the expected maximal profits of substitute goods, which are different in their unit purchase cost and in their shelf-life duration, and decide which product is more profitable, given consumers' sensitivity to freshness. Furthermore, the model allows us to calculate the loss of profits to a retailer who ignores the decrease in the demand rate due to loss of freshness, and uses only a price-dependent demand function (as common in the literature).

The rest of this paper is arranged as follows: In Section 2 we review the literature on the effects of pricing policy and of time on the demand rate, and we position our model in relation to previous work. Section 3 details the model assumptions and notations. In Section 4, we formulate the model as a profit maximization problem, and in Section 5, we find the optimal price for a given replenishment period. In Section 6, we present a thorough analysis of the profit function and properties of the optimal solution. A detailed algorithm is provided to obtain the optimal pricing and inventory policies efficiently. In Section 7, numerical examples are given to illustrate the model, followed by a sensitivity analysis. We sum up with a discussion of this work and suggest directions for future research.

2. Literature review

The literature dealing with pricing policy in inventory models began with Whitin (1955) and was subsequently developed by Mills (1959), Karlin and Carr (1962) and Zabel (1970, 1972). The latter studies refined Whitin's formulation by specifying demand parameters as a function of the selling price, and they were further investigated in Young (1978), Polatoglu (1991), Petruzzi and Dada (1999) and recently in Guowei et al. (2012). Important works on pricing in the context of inventory of perishable items appear in the literature reviews of Silver (1981), Nahmias (1982), Weatherford and Bodily (1992) and Gallego and van Ryzin (1994). Chun (2003) considered dynamic price adjustments when the fixed order cost and holding costs are ignored; his approach departed from that of most previous works, which assumed that price is determined at the start of the sales period and remains fixed. In a recent work, Herbon et al. (2012) captured the main characteristics of the effect of price differentiation in a stochastic deteriorating inventory with heterogeneous consumers.

The literature dealing with time-dependent demand rate mostly focuses on cases in which the influence of time on demand is measured relative to a specific point in time, such as the introduction of a new product into the market (e.g., Bahari-Kashani (1989), Bose et al. (1995), Benkherouf (1995), Teng (1996), and Balkhi and Benkherouf (2004)). Most studies addressing the effect of the time elapsed from replenishment on demand rate have done so indirectly, such as in the case of inventory-level-dependent demand, which can be converted (by solving a differential equation) to a dependency of the demand rate on the order size and time after replenishment. Examples of inventory-level-dependent demand models appear in Urban (2005), who presents a literature review of deterministic models, Wu et al. (2006), who extended the model to include non-instantaneous deterioration of items, and recently in Devangan et al. (2013), who extended the model to stochastic demand. A different approach is used by Avinadav and Arponen (2009) who considered directly the freshness effect of products' remaining shelf-life duration on the demand rate.

Recent works, such as You and Hsieh (2007) and Chang et al. (2010), suggested models that incorporate both inventory-level and price effects on the demand rate. However, there is a paucity of (direct) research on the combined effects of both price and time after replenishment on the demand rate of a perishable item. To our knowledge, only three studies have combined these two effects. You (2005) assumed an additive influence of the two factors, studying a case in which the price effect is linear and the time effect is negative-exponential. Valliathal and Uthayakumar (2011) assumed a general multiplicative influence of these two factors when demand is convex in price. They introduced an algorithm to obtain optimal pricing and replenishment policies; however, their algorithm, which required solving iteratively a system of three equations, does not guarantee convergence towards the optimum. Maihami and Kamalabadi (2012) assumed a special case of multiplicative influence of these two factors, where price has a linear effect and time has an exponential effect.

Our study specializes in perishable items with a finite expiration date (at which demand vanishes). In order to capture this property, and since in many real-life cases only part of the inventory is presented to the consumers (while part of it is stored in a backroom warehouse), we assume a negative influence of the time following replenishment on the demand rate and not a positive influence of the inventory level as used, for example, by Wu et al. (2006) and Devangan et al. (2013). Specifically, we assume a multiplicative effect of price and time on the demand rate, which implies that a change in one factor has a fixed relative (percentage) effect, for any value of the other factor. Similarly to You (2005) and Maihami and Kamalabadi (2012), we assume a linear effect of the price on the demand rate, and, like Avinadav and Arponen (2009), we assume that the demand rate decreases polynomially over time following replenishment, until it vanishes at the expiration date of the perishable item (which is known to the consumer). The polynomial function is more flexible (can be convex, concave or linear) than the convex exponential function (used, e.g., by You (2005), You and Hsieh (2007), Maihami and Kamalabadi (2012)), and thus may fit better to modeling the demand for perishable items with a known expiration date (see next section). In our model, beyond the expiration date of the product, the "effective demand" (i.e., the willingness to buy it) is zero regardless of the retail price or the quantity of leftover units, although the demand for fresh units of the product (i.e., the "potential demand") always exists.

Our model is distinct from previous works in reducing the three-variable optimization problem into a single-variable one, and in providing an efficient line-search algorithm that is guaranteed to obtain the optimal solution. Specifically, we develop a closed-form expression of the optimal price and order quantity, given the replenishment period, and we prove that the profit function is pseudo-concave over a bounded interval of the replenishment period.

3. Assumptions and notation

Consider an inventory system of a single perishable item that is periodically being replenished over an infinite planning horizon with an ordering set-up cost. Here below, we summarize the assumptions of the model.

1. Replenishment rate is infinite with a fixed lead-time.
2. Backlogging is not possible.
3. All items in an order have identical shelf-life duration that is known to the consumers.
4. There is no demand for items that passed their expiration date.